

AN
INTRODUCTION
TO
NATURAL PHILOSOPHY.

ILLUSTRATED WITH COPPER PLATES.

By ~~WILLIAM~~ NICHOLSON.

Non enim me cuiquam mancipavi, nullius nomen fero. multum magnitudo vultus iudicio creco, aliquid et meo vindico Nam illi quoque, non inventa, sed quærenda, nobis reliquerunt
SENECA.

THE FOURTH EDITION, WITH IMPROVEMENTS.

IN TWO VOLUMES.

VOL. I.



LONDON.

PRINTED FOR J. JOHNSON, NO. 72, ST. PAUL'S CHURCH-YARD.

MDCCKCVI.

TO

SIR JOSEPH BANKS, BART.

P. R. S.

SIR,

GRATITUDE and respect are due from every individual in society, to him who promotes its real interest, by extending the bounds of knowledge. All Europe is acquainted with the exertions you have made for its advancement: permit me to join the voice of nations, by expressing the sense I entertain of them. With this intention, I take the liberty of dedicating the following Treatise to the President of that respectable Body of Men, among whom the

VOL. I.

A

true

true Philosophy had its origin, and to whom
it owes a great part of its improvements.

I am, with much respect,

Sir,

Your most obedient

humble Servant,

London,
Feb. 28th, 1782.

W. NICHOLSON.

P R E F A C E.

THE advantages derived from the Science of Natural Philosophy, are so great and so universally acknowledged, that an enumeration of them would be unnecessary, if it did not serve to enliven and direct that spirit of inquiry which is natural to youthful minds; and to awaken those who from a want of reflection are not inclined to look into the causes of things. We are apt to regard objects to which we have long been familiarized, with languor and indifference, and we now behold effects without even the emotion of curiosity, which in less enlightened ages would have been thought miraculous.

Man in a rude and savage state, with a precarious subsistence, exposed to the inclemencies of the seasons, and the fury of wild beasts, is an object of pity when compared to man enlightened and assisted by Philosophy: Ignorant of architecture, of agriculture, of commerce, and of all the numerous arts which depend upon the mechanic powers; he exists in the desert, comfortless and unsocial, little superior in enjoyment to the lion or the tyger, but much their inferior in strength and safety. If it be true that man ever existed in this state, it could not have lasted long; the exertion of his mental strength must

A 2

have

have given life to the arts. Aided by these, the wilderness becomes a garden embellished with temples, palaces, and populous cities; and he beholds himself removed to an immense distance from the animals, to which in his original ignorance he seem-

ed to be united. These sciences bestow that leisure and independence which have enabled superior minds to form laws, and to establish the rights of mankind by mutual compact between the powerful and the weak. By this leisure it is that ingenious and speculative men have collected masses of knowledge, which induce us to regard the powers of the human mind with astonishment. Hence we possess the admirable science of Astronomy. A science founded on the most accurate and long continued observations, and systemised by the purest mathematical reasoning; but at the same time so remote from vulgar apprehension, that its daily and important uses and predictions are hardly sufficient to prevent its being regarded by the ignorant, as a chimera!

The other departments of Natural Philosophy, are not less replete with wonders. How great would have been the surprise of the ancients, could they have foreknown the effects which are produced by the reflection and refraction of light! By a skilful management of these properties, telescopes and various optical instruments are constructed: objects too remote to be perceived by the naked eye, are enlarged and rendered visible: the satellites of
Jupiter

Jupiter and Saturn, the mountains and valleys in the moon, and the changes which take place in the sun's disc, are thus discovered, and afford matter for admiration and enquiry.

Neither is this delightful science of optics confined to the contemplation of distant objects. Minute animals, the vessels of plants, and, in short, a new world in miniature, is disclosed to our view by the microscope, and an inexhaustible fund of rational entertainment and knowledge is brought within the sphere of our senses.

Every one is acquainted with the benefits derived from the science of Hydrostatics, to which we are indebted for many useful inventions. Among these are wind and water mills, pumps, fire engines, steam engines, &c. &c.

Chemistry is productive of great and singular advantages to society. Metallurgy in its utmost extent, the arts of making glass and pottery, of dyeing, and many others, together with a very considerable part of the Materia Medica, are dependant on this branch of Philosophy. The vast importance of Metallurgy, may be rendered obvious from the single consideration of the many uses to which iron is applied. Without this metal we should be almost totally incapable of making any useful or instrument. It is difficult to recollect any production of art in the formation of which iron is not made use of, and the very existence of naval commerce depends on its magnetical property.

Astronomy is not therefore a dry study, but a pursuit of the highest utility and entertainment. Those who cultivate the sciences know that they naturally produce a sincere and disinterested love of truth. An enlarged view of things destroys the effects of prejudice, inspires the properest ideas of the great original cause, and promotes a detestation of every thing that is mean or base. And if there be a pleasure in attending to objects which fill the mind by their immensity, and delight the imagination by the continual discovery of new and sublime analogies, it is not to be wondered that Philosophers pursue their studies with a degree of attention and ardor, which is not found in any other set of men.

The order of arrangement in the present work, is such as was suggested by the subjects themselves. After a cursory enumeration of the general properties of matter, motion is principally attended to, being that affection of matter by which all changes are brought about. Mechanics and astronomy naturally follow, and are succeeded by an elucidation of the properties and motion of light. The more complex motions of fluids, and the atmospheric phenomena are next considered. Thus far it will be observed, that the work treats of such general effects as arise from the motions of bodies, without any particular respect to those specific properties which distinguish them into various classes. The remaining part of the treatise is employed upon these specific properties: a long section upon chemistry is given for the purpose

purpose of explaining them as far as they are as present known, and are capable of being understood by mere reading. Magnetism and electricity occupy the concluding sections. Upon the whole, therefore, it will be seen that the most scientific and best established parts of Natural Philosophy are first treated of, and are followed in succession by others which are less understood.

This treatise being intended to give a clear account of the present state of Natural Philosophy, to such as possess very little mathematical knowledge; care has been taken to select such facts and experiments as tend to establish elementary truths. The varieties of experiments of the same kind are not therefore numerous; but it is hoped that the advantage of a greater number of general principles, is by that means obtained. Philosophical instruments likewise, are not minutely described. References to the parts of drawings are not often read or understood: for which reason it was thought better to explain their general construction, and leave the minutiae to ocular inspection. The grand object throughout has been to relieve the memory, and assist the understanding, by conciseness and illustrative arrangement.

Those prolix disquisitions, which render the commentarior less intelligible than the author commented upon, are thus avoided; neither has the affectation of familiarity, which is usually attended with a lax,

and unphilosophical explanation of one event by another equally obscure, been indulged. On the contrary, the author has every where endeavoured to preserve that solidity of argument, and precision of expression, which distinguish the works of the best Philosophers. And, notwithstanding the nature of the undertaking unavoidably required a deviation from those elegant and general principles which are obtained by strict mathematical reasoning; yet it is presumed that the student will find nothing in this treatise, which he will be under the necessity of unlearning, when he attempts the perusal of those books to which this is offered as an introduction.

The attentive examination of other books, to which the writer of this performance has had recourse, has shewn him, that even the works of those great men, who deserve and possess the highest reputation, are not free from errors of importance. The present occasion does not require the disagreeable task of pointing them out; but this very consideration will not permit him to hope that his diligence has entirely excluded mistakes. However he has little to fear on that account, being sensible that those who are the best able to discover them, will at the same time be the readiest to exercise that candor which every writer has need of.

The liberty which has been taken in altering the words of other authors, and adapting them to the purpose of this work, would have prevented the use
of

of formal quotations if they had been supposed necessary; and as the present intention is not at all historical, the names of authors have been avoided as much as was consistent with the wish of the writer, to evade the suspicion of plagiarism. If plagiarism can be imputed to the author of an epitome of science, this acknowledgment must be allowed to obviate the charge.

In the printing every thing which could be imagined of service to the book, as a manual of philosophy, has been done. A varying title at the head of each page, and copious indexes, are annexed. From these the reader will see that scarcely any fact of importance has been omitted.

The learner who may be induced to fix his chemical reading in his memory by recurring to experiment, which may be done with very little expence, is cautioned to beware of the danger with which it is sometimes attended. The solution, evaporation, and calcination of uninflamable matters, may be performed in the common apartments of a dwelling house; but the distillation of corrosive or inflammable substances, ought not to be attempted but in a place prepared for the purpose. The bursting of a retort containing any concentrated fuming acid, must be very destructive to furniture, as well as prejudicial to health; and ardent spirits, resins, and the like, would endanger the house if a similar accident were to happen. It is impossible to give
advice

advice against the many casualties to which chemical experiments are liable. One general maxim is always to endeavour from analogy to foresee the consequence, or probable result, of the intended process, and when that cannot be done, to observe the phenomena, and proceed with caution.

ADVERTISEMENT

TO THE

SECOND EDITION.

THE improvements in this second edition are very considerable; as well in consequence of the rapid progress of the discoveries made by Philosophers in all parts of the civilized world, as of the careful revision the whole work has undergone. The additions are equivalent to a third volume, though by an alteration of the type, and page, this edition has been prevented from exceeding the former in bulk or price. Many additional figures have been inserted in the plates, and those numbered vii. and viii. are entirely new. Marginal references are likewise annexed, which it is hoped will be found eminently useful. In these the figures denote the pages, and the letters the paragraphs of the pages, where the proofs or illustrations referred to are to be found: and in the second volume the numeral letter i. is

is prefixed whenever the first volume is referred to.

London,
Nov. 1st, 1786.

Of the present fourth edition I have only to observe, that the whole has been carefully revised, and the recent discoveries added in their proper places.

London,
January 8th, 1796.

CONTENTS.

INTRODUCTION.

	Page
Of the degrees, or kinds of knowledge, and the rules of philosophizing	1

BOOK I. SECT. I. *Of Matter in the Abstract.*

CHAP. I. Of matter, and its properties	7
II. Of extension and impenetrability	9
III. Of the inertia, and motion or rest	18
IV. Of attraction, considered chiefly as a power that generates motion	25
V. Of the production, communication, or destruction of motion	33
VI. Of the attraction of cohesion, and of specific attractions	44

BOOK I. SECT. II. *Of Bodies in Motion.*

I. Of the mechanical powers	53
II. Of the lever	55
III. Of the axis and wheel, and of the pulley or tackle	60
IV. Of the inclined plane, and of the wedge	64
V. Of the screw, and of mechanical engines in general	68
VI. Of the center of gravity	74
VII. Of pendulums	80
VIII. Of the motion of a body, which is acted upon by a centripetal force.	95

BOOK I. SECT. III. *Astronomy.*

	Page
CHAP. I. Concerning the system of the universe	101
II. Of the figure and motion of the earth	109
III. Of the mutual appearances of the superior and inferior planets	113
IV. Of the superior planets, and of the true form of the planetary orbits	118
V. Of the affections of the planets	124
VI. Of parallaxes, and of the transit of venus	128
VII. Of the secondary planets	137
VIII. Of the moon	141
IX. Concerning the eclipses of the sun and moon	145
X. Of comets; and of the proportion of light and heat on the planets	159
XI. Of the telescopic appearance of the moon	167
XII. Of the telescopic appearances of the sun and the planets	174
XIII. Of the length of days and nights, and of the seasons	180
XIV. Of the fixed stars	189

BOOK I. SECT. IV. *Concerning the Physical Causes of the
Celestial Motions.*

I. Of the general effects of centripetal forces on bodies in motion	197
II. The universality of gravitation deduced from its effects	207
III. Of the irregularities arising from the mu- tual gravitation of the planets	218

CHAP. IV. Of the figures of the planets; the precession of the equinoxes, and the nutation of the earth's axis	234
V. Of the sides	241

BOOK II, SECT. I. *Of Light.*

I. Concerning the motion of light	253
II. Optical definitions and principles	261
III. Of the various refrangibility of the rays of light	264
IV. Of the various reflexibility of the rays of light	270
V. Concerning the rainbow	273
VI. Of the separation of the original rays of light by reflection or transmission, that depends on the thickness of the medium upon which they are incident	280
VII. General inferences respecting the disposition to be reflected or transmitted, into which the rays of light are put, by the action that depends on the thickness of the medium upon which they are incident	287
VIII. Of the permanent colours of natural bodies, and the analogy between them and the colours of thin transparent plates	292
IX. Of the inflections of the rays of light which pass in the vicinities of bodies	298
X. Of the powers by which bodies reflect or refract the rays of light	304

BOOK II. SECT. II. *Of Optics.*

I. Concerning the reflection and refraction of light by surfaces regularly formed	315
---	-----

CONTENTS.

	Page
CHAP. II. Of dioptrics; or the regular refraction of light	322
III. Of the eye; and of vision	327
IV. Of refracting microscopes; or the dioptric instruments, by means of which small and near objects are magnified	335
V. Of refracting telescopes; or the dioptric instruments, by means of which remote objects are rendered large and distinct to the view	342
VI. Of the imperfections of telescopes, and their remedies; and of the achromatic telescope	348
VII. Of catoptrics, or the regular reflection of light; and of the reflecting telescope	354

(1)



AN
INTRODUCTION
TO
NATURAL PHILOSOPHY.

OF THE DEGREES OR KINDS OF KNOWLEDGE; AND
THE RULES OF PHILOSOPHIZING.

THE impressions made on the organs of sense by external objects produce ideas in the mind. We are continually employed in amassing a stock of general truths respecting them, which is called knowledge.

An intelligent being, whose powers are limited, must of necessity be unequal to the performance of many things. If any assertion be made respecting two or more ideas, it will imply either truth or falsehood; but the cases in which we can with certainty discover the one or the other are very few, in comparison with those that are placed beyond our power of investigation. Yet, as a decision is almost always required for the regulation of our conduct in the affairs of life, the greater part of our knowledge becomes founded on probability, instead of established truth. The means of acquiring knowledge may therefore be said to be of three

Vol. I. B kinds.

kinds. Certainties are obtained either by intuition or demonstration; probabilities are obtained by analogy.

There are some ideas whose mutual relation in certain respects is so evident, that nothing more is required to obtain the knowledge of it, than to apply them to each other. For example; if a given body be divided into parts, and the mutual relation between the whole body and one of its parts, with respect to magnitude, be demanded, the mind immediately conceives, with the clearest and most absolute certainty, that the whole body is greater than its part. If the particular body or magnitude in contemplation be abstracted, or left out, the proposition becomes general in this form, viz. every magnitude is greater than any part of the same. This kind of knowledge is called intuitive, and the general propositions are termed Axioms.

When it is required to determine the mutual relation of two ideas, whose agreement or disagreement cannot be intuitively perceived, the truth may often be obtained by the interposition of a chain of axioms. This method of exhibiting the truth is termed demonstration, and seems to be applicable only to our ideas of the quantities and positions of magnitudes. For this reason, it will be difficult to give an example without having recourse to the mathematicks. The following will, however, be easily understood.

Fig. 1. Let the two circles in the figure be supposed equal, and the circumference of each to pass through

through the center of the other. Imagine the centers to be joined by the right line AB , and the lines CA , CB , to be drawn from one of the points where the circumferences intersect each other, to the centers respectively. Then, I say, the lines AB , BC , CA , will be equal each to each.

The demonstration of this truth is as follows:

The word circle signifies a plain figure, contained under one line, called the circumference, to which all right lines drawn to a certain point within the figure, called the center, are equal. As soon therefore as it is understood that the figure ACD is a circle, and that the lines AB , CB , are right lines drawn from its center to its circumference, it is acknowledged intuitively, and without further argument, that those lines are equal.

The same reason in the circle ECB evinces, that the lines AB , AC , are equal.

The lines AC , CB , being thus proved to be each equal to the line AB , are likewise equal to each other. For it is an intuitive truth or axiom, that things equal to one and the same thing are equal to each other.

The want of axioms, and the labour of demonstration, are not the only impediments to the acquisition of knowledge. Knowledge is conversant with ideas only: it can therefore be said to possess reality with respect to eternal objects, so far only as those ideas may be taken or substituted for the things they represent; but it is impossible to determine how far this may be done with propriety, even if it can be

done at all. In referring from ideas to things we are liable to error, not only because the compound idea of a being consists of an assemblage of its properties, which may be incomplete and inadequate, but likewise because those ideas may even be quite different from any thing existing in the being itself, as may be instanced in the ideas of colour, sound, pain, &c. The great perspicuity and certainty of mathematical knowledge arises from the simplicity of the ideas employed, and their not depending on any external being: for, as this science treats only of ideas, it is of no consequence to its truths, whether geometrical figures ever had an existence; it being sufficient that their existence is possible.

The greater number of our ideas being too complex and imperfect to admit of intuitive conclusions or axioms, it is evident, that in general we must be contented with less proof than demonstration. Instead therefore of endeavouring to obtain axioms by comparing ideas, we observe events, and from the contemplation of what has happened, we form a presumption of what will again come to pass. Observation has shewn us, that a certain event is always followed by another determinate event; we suppose a relation to subsist between them; we imagine this relation to be necessary; we distinguish the prior event by the name of Cause, and the latter we call the Effect. This kind of knowledge, which is not founded on reasoning, but on experience alone, may be termed Analogical, and is much less perfect than what is obtained by intuition or demonstration.

monstration. That a stone will descend to the earth, is an analogical proposition. It cannot be demonstrated; but, from the consideration of a vast number of events of the same nature, a degree of probability arises, which commands our assent. It is clear, that analogical propositions are no more than strong probabilities, from the remarkable circumstance, that their converse does not imply an absurdity. To deny an intuitive or demonstrative truth, is to assert an impossibility; but to deny an analogical truth, is only to assert an improbability. The understanding revolts at the affirmation, that a part is greater than the whole body; but we see no impossibility in the assertion, that a stone, at some time or place, has remained in the air without a tendency to descend; this supposition being highly improbable, but nothing more. In fact, demonstration is a collection of truths or axioms; analogy is a collection of probabilities. Simple probabilities are to analogy what axioms are to demonstration. Now, there is no comparison in point of certainty between axioms; all being equally true; but probabilities differ exceedingly in their degree of credibility.

Natural Philosophy, strictly speaking, admits of no other proofs than those of analogy. To give stability to this science, it is necessary to admit no probabilities as first principles of analogy, but those which possess the strongest and most incontrovertible resemblance to truth. For this purpose, the following rules are adopted:

Rules of Philosophizing.**I.**

No more causes of natural things ought to be admitted than are real, and sufficient to explain the phenomena.

II.

And therefore effects of the same kind are referred to the same causes.

III.

Those qualities, whose virtue can neither be increased nor diminished, and which are found in all bodies with which experiments can be made, ought to be admitted as qualities of all bodies in general.

B O O K I.

S E C T. I.

Of Matter in the Abstract.

C H A P. I.

OF MATTER AND ITS PROPERTIES.

MATTER is known to us only by its properties. A

The properties common to all matter are extension, impenetrability, inertia or resistance, attraction, motion, and rest; all which, except the two last, which cannot exist together, are found in all bodies whatsoever. B

It would be, perhaps, a fruitless attempt, to enquire whether these are the only qualities with which bodies are endued in common. Matter may possess many others, that our senses are not adapted to observe, or which have even escaped the notice of Philosophers. But it is necessary to observe, that we are totally ignorant of the substance in which these properties are united. The essence of matter is unknown to us. We must not, therefore, assume one or more of these properties as composing that C

essence itself; for most of the errors of the earlier philosophers have arisen from this source.

- D There are other properties, sometimes called specific, that are not found in all bodies; such as transparency, opacity, fluidity, consistence, and the like. But these seem to relate to the figures or motions of the parts of bodies, and are, therefore, referable to the general properties. There are also several species of attraction and repulsion, which will be attended to in their proper places.

Here follow definitions of the general properties above mentioned.

- E Extension, is that affection of matter by which it occupies part of space.
- F Impenetrability, is that by which two bodies cannot exist in the same place at the same time.
- G Inertia, is that by which a body resists any force impelling it to a change of state, with regard to motion or rest.
- H Attraction, is that by which one body continually tends to approach to, and, if not by external means prevented, does approach to, another body or bodies.
- I Motion, is a continual and successive change of place. Rest, is the permanency or remaining of a body in the same place.

C H A P. II.

OF EXTENSION AND IMPENETRABILITY.

THE idea of extension is so simple, that it cannot be defined. For though in the preceding Chapter it was pointed out as that affection by which matter occupies part of space, yet, there can be little doubt but that the idea of extension is itself antecedent to that of space, and therefore not properly definable by it. In order to facilitate the consideration of such truths as relate to extended magnitudes, geometers have, as it were, analyzed extension. It is evident that extension implies form or figure, and figure must be limited. The limit or termination of figure is called a surface, or superficies. A superficies is likewise limited, and its termination is called a line. And the termination of a line is called a point. Now, though it is clear, that a superficies, a line, or a point, cannot exist separate or apart from an extended being, yet, it is certain that the ideas of them may be considered distinctly, without immediately referring to the other consequences arising from the general idea of extension. In this sense mathematicians define a point to be that which has no part, or is altogether indivisible; a line to be that which is length, without breadth, or is divisible in one respect, namely, of length; a surface to be that which has only length and breadth, or is divisible in two respects, namely, of length and breadth; and a solid

to be that which has length, breadth, and thickness, or is divisible in three respects, namely, of length, breadth, and thickness.

o No finite or imaginable division of a line can ever produce a point or indivisible part. A line is therefore divisible into an infinite number of other lines, or similar parts, and consequently much more is a superficies, and yet more a solid. A mathematical solid, that is to say, pure extension, is divisible to infinity : and if the elements of matter be of the same nature as the aggregates they compose, matter is likewise infinitely divisible. A variety of remarkable consequences follow from these plain deductions ; of which we shall proceed to mention a few instances both theoretical and practical. Thus,

p Any quantity of matter, how small soever, and any finite space, how great soever, being given (as for example, a cube circumscribed about the orb of Saturn) it is possible for the small quantity of matter to be diffused throughout all that space, and to fill it, so that there shall be no pore or interstice in it, whose diameter shall exceed a given finite line.

To prove this, suppose the cube to be divided into small cubes, whose sides shall each be equal to half the given line. It will be easily seen, that the number of small cubes will not be infinite. Imagine the quantity of matter to be then divided into a number of parts equal to that of the small cubes, and a particle to be placed in the center of each cube. The whole space will thus be filled, and the

the greatest distance between two adjacent particles, or, in other words, the diameter of any pore or interstice, will be less than the given line.

Hence there may be given a body, whose matter, if it could be reduced or compressed into a space absolutely full, that space may be any given part of its former magnitude.

Again; there may be two bodies of equal bulk, *Q* and their quantities of matter may be unequal in any proportion; yet the sum of their pores, or quantity of void space in each of the two bodies, shall be nearly in the proportion of equality to each other.

This is not so obvious as the former instance; but an example will render it clear.

Suppose one thousand cubic inches of gold to contain one cubic inch of matter, or, in other words, when reduced into a space absolutely full, to be equal to one cubic inch: then one thousand cubic inches of* water will contain one nineteenth part of an inch of matter when reduced. Consequently, the void spaces in the gold will be nine hundred and ninety-nine cubic inches, and those in the water nine hundred and ninety-nine cubic inches, and eighteen nineteenth parts of an inch; that is, they will be nearly in the ratio of equality.

Yet, the actual divisibility of matter can proba- *R*
bly be carried but to a certain degree. The ultimate particles of bodies, it is most likely, are not to be altered by any force in nature. But there are, nevertheless, many instances which shew to

* Gold is nineteen times as heavy as water.

what inconceivably minute parts bodies may be actually divided.

- s A grain of leaf-gold will cover fifty square inches; and contains two millions of visible parts; but the gold which covers the silver wire, used in making gold lace, is spread over a surface twelve times as great.
- T In making this wire, it is usual to gild a cylindrical bar of silver strongly, and afterwards draw it into wire, by passing it successively through holes of various magnitudes in plates of steel. By this means the surface is prodigiously augmented, notwithstanding which, it still remains gilded so as to preserve an uniform appearance even when examined with the microscope. The quantities of gold and silver, and the dimensions of the wire, are known. With these data it is easy to calculate, and from calculation it is proved, that sixteen ounces of gold, which, if in the form of a cube, would not measure one inch and a quarter in its side, will completely gild a quantity of silver-wire sufficient to circumscribe the whole globe of the earth.
- U The animalculæ observed in the milt of a cod-fish are so small, that many thousands of them might stand on the point of a needle.

Supposing the globules of the blood in these animalculæ to be in the same proportion to their bulk as the globules of a man's blood bear to his body, it appears, that the smallest visible grain of sand would contain more of these globules than

10,256 of the largest mountains in the world would contain grains of sand.

These instances may serve to shew the amazing v fineness of the parts of bodies, which are nevertheless still compounded. Gold, when reduced to the thinnest leaf, still retains those properties which arise from the modification of its parts. Microscopic animalculæ are without doubt organized bodies, and the particles of their circulating fluids must be possessed of specific qualities. Even the rays of light are compounded of an almost infinite variety of particles, which, when separated from each other, exhibit the powers of exciting ideas of colours. None of these are the ultimate particles of which all bodies are formed, for they all bear evident marks of composition. How inconceivably small then must those particles be!

To these ultimate particles alone it is, that impenetrability can be attributed. Penetration takes place in all compounded bodies. Water exists in the pores of wood. Air in the pores of water. Quick-silver in the pores of gold, &c. &c.

Some philosophers have questioned whether impenetrability be really a property of matter; and it must be confessed, that, notwithstanding this idea is so closely connected in the formation of our compound idea of matter, yet, if we examine from whence the notion is originally obtained, we shall find that our knowledge is much less certain than we may have suspected.

To make this clearer, we must consider that our notion of impenetrability is derived from the sense
of

of feeling. We move the hand towards a body, and in situations where motion is not generated, it is prevented by that body from going forward; from which we conclude, that the body possesses a part of space to the exclusion of every other body; that is to say, that it is impenetrable.

- z But, in order to justify this conclusion, it is necessary that we should be certain that it is the body itself, actually occupying space, which resists the pressure; and of this we cannot be assured, since we observe many instances in which bodies afford resistance to other bodies which move in spaces at some distance from the resisting body. Thus, the loadstone, in certain circumstances, resists the motion of iron which approaches towards it; and there is no doubt but this resistance or repulsion, if exerted on any part of a man, would afford a sensation similar to that which arises from contact. If the man had not sight, or some other sense to perceive that the resisting body was really distant, he would, from the sense of touch, conclude that the body was in contact with the part perceiving; and, if any force he could produce were insufficient to overcome that resistance, he would conclude the body to be impenetrable.

Now, by several experiments, which we shall have occasion to mention in the course of this work, there is the highest reason to conclude, that all bodies exert a repulsive force on each other, and that the common effects which are attributed to contract and collision are produced by this repulsion:

sion: And, if so, why not attribute all effects of the same nature to this cause, which we know exists, instead of supposing an impenetrability that can never be proved?

If the force of repulsion be sufficiently great, it may not be in the power of any natural agent to overcome it, and, consequently, all the effects of a real impenetrability will take place, though the substance or matter itself may not be impenetrable, or even extended.

It is not in our power to determine, whether impenetrability or extension be essentially necessary to existence. For the extension of the elements of matter seems capable of no other proof than what may be drawn from their impenetrability; and experiment cannot decide, because a finite pressure can only prove the reaction of a finite resistance.

The question therefore is, whether it be more probable, that the particles of matter are beings possessed of a finite power of repulsion, which prevents their mutual approach, but does not render mutual penetration or coincidence in the same part of space impossible, on the application of force sufficient to overcome that repulsion; or whether they be impenetrable atoms, which, consequently, must resist such coincidence with an infinite force?

Here we must attend to the facts. If the repulsion continually increased as the distance of the bodies decreased, we might conclude that it was the only cause of the apparent impenetrability of bodies;

bodies; but, as in the loadstone, there is a certain small distance at which repulsion ceases, and attraction takes place, so in compressing bodies together, with a certain degree of pressure, the distance is at length diminished sufficiently for the bodies to adhere. The phenomena are probably similar; but, at all events, the cohesion of the parts of bodies shews a mutual attraction; and it is not easy to explain why the parts should not mutually penetrate and coincide, when the repulsion on which their impenetrability was supposed to depend has ceased, and given place to attraction. And on this account the doctrine of impenetrable atoms would seem the most probable.

✓ This deduction, however, supposes the impenetrable particles to come into contact, where attraction has taken place. But it is certain they do not. For things in absolute contact cannot come nearer without penetration, and cold is known to diminish the bulk of bodies; or, in other words, to bring their parts nearer each other than before. This contraction is greater, the greater the cold. Without enquiring into the nature of cold, we may therefore presume that the utmost possible cold would either bring the impenetrable particles into absolute contact, or cause the body to vanish by the mutual penetration of all its elements. Thus, the original question returns to us, and the only remaining argument seems to be——If by the first rule of philosophizing (6) we are to admit no more causes of natural things than are sufficient to explain the phenomena,

phenomena, and we know that a sphere of repulsion exists as the proximate cause of our ideas of impenetrability and extension, why should we add to this an extended atom existing in the center of the sphere of repulsion?

The quantity of matter in the universe is much **o** less than is generally imagined. This truth may be deduced from what has been said already on this subject; but more especially from the properties of transparent bodies. Light passes through these in all directions without the least difficulty. The focus of a burning mirror, which augments the density of the sun's rays upwards of three thousand times, may be received in the bodies of glass or water, without producing any effect; so far are the particles of those substances from impeding the passage of light. And the bottom of the sea has been discerned at a greater depth than sixty feet. It seems not improbable that the real matter in a small piece of glass may bear a less proportion to its bulk than that bulk does to the whole earth.

To render the possibility of this more evident, **n** we may suppose a body to be so constructed, as to have as much vacuity as matter; then half the body would be vacuous. And if the particles of which it is composed, be constructed in the same manner; then the vacuity will become three-fourths of the space occupied by the body. Again, let these last mentioned particles be constructed in the same manner; the vacuity will then be seven-eighths.

And if the series be carried forward to the tenth order of particles, the vacuity will exceed the matter one thousand and twenty-three times.

C H A P. III.

OF THE INERTIA, AND MOTION OR REST.

- I It is chiefly from the inertia that we obtain a knowledge of the relative quantities of matter in bodies. The quantities of matter in bodies absolutely similar in composition, are determined by their extension; but in dissimilar bodies the proportion does not hold. Now, in bodies similar in composition, we observe that the inertia follows the proportion of the extension, that is, by reason of the similarity, the proportion of the mass of matter; and from thence, by applying the proportion of the inertia to dissimilar bodies, we obtain a knowledge of their masses. Thus, for example, the quantity of matter in one cubic inch of gold is as 1, in two cubic inches as 2, in three cubic inches as 3, and so forth: this we gather from the extension, and also from the inertia, both which, in this case, follow the same proportion. But if a cubic inch of copper be added, though the extension be augmented as 1, yet the inertia increases only as $\frac{1}{2}$; therefore, either the extension or the inertia is not the proper measure of the mass; and, as we can more readily admit that the extension,

extension, or space occupied within the external limits of the body, may, by porosity in the body, cease to be the measure, than that the inertia of the ultimate parts of matter should vary; we conclude, that the quantity of matter is as the quantity of the inertia; though it must be allowed that neither position is physically demonstrable.

The inertia of matter being that by which it **K** resists any change of state with regard to motion or rest, is measured by the force which is required to produce a given change; that is to say, the force required to give a certain degree of velocity to a body at rest **A**, containing ten parts of matter, is five times as great as would produce the same effect on a body at rest **B**, containing two parts.

This force in a moving body is called the quan- **L** tity of motion, or momentum, and is measured by the mass of matter multiplied by the velocity; for the whole motion of a body is the sum of the motions of all its parts. Therefore, in the last mentioned instance, the body **A** moves with five times the force that **B** moves with, though the velocity is the same in both. But if the velocity of **B** were to be augmented five times, the quantities of motion would then be equal; that of **A** being expressed by 10, multiplied by 1, and that of **B** being expressed by 2, multiplied by 5.

Motion and rest are distinguished into absolute and relative. Absolute motion is the removal of a **M** body from one part of space to another. Relative **N** motion is a successive change of situation with

respect to another body, though that body may not be at rest. Thus, a man sitting in a barge in motion, is relatively at rest, that is, with respect to the parts of the barge: but absolutely in motion; being removed, with the vessel, from one part of space to another. On the contrary, the bargeman, who fixes a staff in the ground, and gives motion to the barge by walking along its gunwale, is absolutely at rest, for the staff against which he leans is fixed; but relatively in motion, since, with respect to the vessel, he walks from one end to the other. But if the earth be supposed in motion, the absolute motion of the barge and its contents will be compounded of its relative motion, together with the absolute motion of the earth.

- o There is another distinction in motion, by which it is called apparent or angular, and which depends on an optical fallacy. Thus, to an eye at *e*, (fig. 2) a body which moves from *c* to *d*, or from *f* to *g*, will apparently describe the line *ab*, though the real motions are very different. And if a body move, either directly towards, or directly from the eye, it will be apparently at rest. It is true, that, from other circumstances, we have acquired the habit of distinguishing different motions which are made obliquely to the eye; but where those circumstances are wanting, as in the heavens, it requires no small degree of attention to distinguish the real from the apparent motion.

Experience

Experience proves, that the three following laws are sufficient to account for all the phenomena of motion.

LAW I.

Every body continues in its state of rest, or of uniform motion in a right line, unless compelled to change that state by forces impressed.

For matter at rest, being endued with no power of moving itself, would remain so for ever, unless impelled by some external cause.

We have also daily proofs, that a body in motion will continue to move uniformly in a right line, unless prevented or diverted by some other agent. The resistance of the air, and the force of gravity, in a short time destroy the motion of all projectiles, which otherwise, by the *vis inertiae*, would continue for ever.

LAW II.

All change of motion is in proportion to the force impressed, and is made in the line of direction in which that force is impressed.

For if any force produces motion, a double force will produce a double quantity, and a triple force a triple quantity, whether it be impressed all at once, or by successive impulses. And this motion, since it always has the same direction as the generating force, if the body be already in motion, either increases the same, by conspiring therewith, or diminishes it by opposition, or is added to it obliquely, being compounded with it according to the directions of the two motions.

LAW III.

- R Action and reaction are always equal and contrary; or, the mutual actions of two bodies are always equal, and in contrary directions.

Thus, if a stone be pressed by the finger, the finger is equally pressed by the stone. If a horse draws a stone, the stone draws the horse equally backwards, for the rope is equally stretched towards both. If one body impels another, it will itself suffer an equal change of motion by the reaction, in a contrary direction. If the loadstone attracts iron, it is itself equally attracted, and both are at rest, when they come together, which could not be if they did not press equally. By this mean the changes of motion, though not of velocity, are always equal.

To illustrate this yet more. Suppose a horse proceeds with a quantity of motion expressed by the number 3, and that it would require a force equal to 2 to move a certain stone. The horse then drawing, proceeds with a force equal to 1, the reaction of the stone destroying as much force as the action communicates to it.

From these laws the following corollaries are easily deduced, which may be applied to solve all the effects which can be produced by the mechanical powers.

COROLLARY I.

- S A body impelled by two forces acting in the direction of the two sides of a parallelogram will describe the diagonal in the same time, as by the action

action of one of the forces, it would have described one of the sides.

A body at A (fig. 3) would be carried with an uniform motion in a given time to B, by the single force M impressed at A; and by the single force N, impressed at the same place, would be carried from A to C: complete the parallelogram $ABDC$, and by the combined forces, it will be carried in the same time in the diagonal from A to D. For since the force N acts according to the direction of the line AC, which is parallel to BD; it will, by Law II. in no respect alter the velocity of approaching to BD, which was produced by the other force. Therefore, the body will in the same time arrive at the line BD, whether the force N be impressed or not; and at the end of the time will be found somewhere in the said line BD. By the same manner of arguing, it will at the end of the time be found somewhere in the line CD, which must of consequence be in that place where they intersect each other. Its motion will be in a right line by Law I.

COROLLARY II.

Hence also appears the composition of a direct force AD, (fig. 3.) from any two oblique forces, AB and BD, and on the contrary, the resolution of a direct force, into any two oblique forces, AB and BD.

The laws of motion being obvious deductions from the phenomena around us, are confirmed by

every mechanical effect we see produced. In the consideration of forces, it is very often convenient to regard them as if compounded of two or more forces, as will be shewn in the ensuing section of this Book. It is clear, that any given motion or force AD may be produced by any pair out of an indefinite number of pairs of forces that may be compounded together; for example, AC and AB , or AF and AE , or, generally, by any two forces, whose quantity and direction may constitute a parallelogram, having AD for its diagonal. In this parallelogram are six things, namely, the directions of the two compounding forces, and of the compounded force, and also their respective quantities. And if any four of these be known, the other two may easily be found, by completing the construction of the figure.

COROLLARY III.

- U** The quantity of motion which is obtained by taking the sum of the motions made in the same direction, or the difference of those made in contrary directions, is not changed by the mutual action of the bodies.

For action and reaction being equal, by Law III. (22, R) will therefore occasion equal changes in the motions, but in contrary directions. Consequently, if the motions are both made in the same direction, whatever is added to the motion of the impelled body must be subtracted from that of the impelling body, and the sum will remain the same.

But if the bodies meet directly, the quantity of motion destroyed being equal in each, the difference of the motions made in contrary directions will remain unchanged.

CH A P. IV.

OF ATTRACTION, CONSIDERED CHIEFLY AS A POWER THAT GENERATES MOTION.

THE force which tends to bring bodies together, without any hitherto discoverable impulse, is called attraction. Whether the various kinds of attraction that fall under our observation may be referable to one common immediate cause, cannot yet be determined. The third rule of philosophizing (6) authorizes us to reckon it among the properties of matter; and whether we shall ever proceed so far as to discover, that it is secondary to some other more remote property must depend on future researches. It seems clear, however, that it is not deducible from any of the properties we have enumerated, (7, 8) and consequently that all suppositions respecting the circulation of effluvia, are mere words without meaning.

The several kinds of attraction are, the attraction of gravitation, or gravity; the attraction of cohesion; the attraction of combination, or chemical affinity; the attraction of magnetism and the attraction of electrified bodies. Whether these be modifications

tions of one simple power produced by variations in the figure, density, temperature, or other secondary qualities of bodies, is entirely unknown. In this place we are to consider only the attraction of gravitation as a power that generates or produces motion.

z Gravitation is that force by which bodies fall to each other. The vicinity of the earth which strongly attracts every thing to itself, prevents its effects between smaller bodies from appearing; but the attraction of the mountain of Schehallien in Scotland, upon the ball of a pendulum, was found by a very accurate set of observations to be considerable.

A This power is found to act on all bodies, at any given place, precisely according to the quantity of matter in each, which is discovered by the vibration of pendulums, thus. Let the two unequal bodies *A* and *E* (fig. 4) be suspended by threads of the same length, and be let fall at the same time from the points, *A* and *F*, in the arcs *AC* and *EG*, which are at equal distances from the two lowest points *D* and *H*. Then the vibrations of each will be performed in equal times, and consequently the velocities will be equal. Whence the quantity of motion in each, being the product of its mass of matter multiplied by its velocity (19, L) will be in proportion to the mass of matter in each. But (21, Q) the force producing motion is in proportion to the quantity of motion produced. Therefore, the force of attraction is in proportion to the quantities of matter in bodies.

This

This likewise appears in falling bodies, all which being let go from equal heights, how different soever in weight, arrive at the ground in the same time, that is, with quantities of motion in proportion to their respective quantities of matter.

The resistance of the air is not here considered, for the sake of perspicuity, though it very sensibly impedes all motions performed in it. A guinea will arrive at the ground in less time than a feather; but in the receiver of an air-pump, out of which the air is exhausted, they both fall in the same time.

Gravitation acts on all bodies at all times, and that equally, whether in motion or at rest, as is evident from the velocities of falling bodies, which are uniformly accelerated during the whole of their course. That a force constantly and equally acting, will produce an uniform acceleration of velocity, is plain, from the following considerations. Suppose a body A, begins to move, by the impulse of gravity impressed at that instant, with a velocity expressed by the number 1, the next instant another impulse will generate a velocity equal to the former. It will therefore move with the velocity 2, and at the third instant with the velocity 3, &c. for the preceding velocities are not in any respect diminished or altered by the succeeding impulses (21, Q). If then the impulses are equal, and equidistant in time, the generated motion will be uniformly accelerated; and the velocity, which in this case may be considered as the motion, for the mass of matter does not alter, will be in proportion to the time; that is, if the velocity

city in 5 instants be expressed by 5, that produced in 10 instants will be 10, &c. This holds good, let the number of impulses in a given time be ever so great. But the number must here be considered as infinite, for the force ceases not to act for the least portion of time, and therefore the acceleration continues uniformly through every part of the motion.

E The space described by an uniformly accelerated motion in a given time, may be conceived to be the sum of an infinite number of spaces produced by a like number of uniformly increasing velocities. These spaces will be as the velocities by which they are described. Therefore, as the sum of the numbers expressing the velocities in any given time is to the sum of the numbers expressing the velocities in any other given time, so is the sum of the spaces, or whole space described in the first given time, to the sum of the spaces, or whole space described in the other given time. But the sums of the velocities, for any terms of time taken from the beginning of the motion, are to each other as the squares of the times.

F For the times uniformly increasing from the beginning, may be expressed by the natural series of numbers, 1, 2, 3, 4, 5, &c. &c. The velocities, and also the correspondent small spaces, may be expressed by the same series, because they are both in the same ratio of the times. The whole time of description, or the sum of the instants, will be denoted by the number of terms, or, which is the same in this series, by the last term. And the whole space described, or sum of the spaces, will be denoted by the sum of the terms.

Now,

Now, all arithmeticians teach, that twice the sum of such a series is equal to the number of the terms added to unity, or 1, and multiplied by the last term. But, because the instants contained in the given time have been assumed infinitely small, the number of terms in the series will be infinitely great, and consequently the addition of unity to the number of terms will not cause any finite augmentation. Rejecting therefore the addition of unity, the double sum of the terms, will be equal to the number of terms multiplied by the last term, or, in this series, to the last term multiplied by itself, that is, the square of the last term; and because halves are as their wholes, the sum of the terms will be in the same ratio. But the sum of the terms expresses the whole space described, and the last term denotes either the whole time, or the last acquired velocity. Whence a the space described from the beginning of an uniformly accelerated motion is as the square of the time, or as the square of the last acquired velocity. Which was to be proved.

If the acceleration cease at the end of any given h time, the motion will become uniform with the last acquired velocity, and the space described in an equal term of time will be double that which before was described from the beginning by the accelerated motion. For the space described in any single instant of the time by the last acquired velocity will be expressed by the last term of the just mentioned series: and the whole space described will be equal to this space multiplied by the number of instants. But the time

or number of instants being equal to the preceding time of acceleration, will be expressed by the number of terms, or, in this series, by the last term. The space described will be therefore expressed by the last term multiplied by itself; that is to say, it will be the double of the space described by the accelerated motion in an equal time.

I When it is said that a number denotes any magnitude, it is to be understood that the number is part of a series, whose terms vary in a ratio always correspondent, or equal to the ratio of the magnitudes denoted; that is to say, the ratio between any two terms of the series is always equal to the ratio between the two magnitudes that correspond to those terms. In this sense, any magnitudes, however unlike, may represent each other. Here follows a geometrical demonstration of the foregoing propositions, in which lines are made use of instead of numbers.

K In the triangle ABC , fig. 5, let the equal divisions $A, 1, 2, 3, \&c.$ on the side AB , represent equal parts of the time of an uniformly accelerated motion. Then the parallel lines, $1d, 2e, 3f, \&c.$ may represent the velocities at the several instants, $1, 2, 3, \&c.$ for they are in proportion to the times $A1, A2, A3, \&c.$ And in like manner for any other part of the time as Am , the velocity generated will be represented by its correspondent ordinate mn . And the sum of the ordinates corresponding with any part of the time will represent the sum of the velocities. But the sum of the ordinates, when taken indefinitely

indefinitely numerous, may be conceived to occupy the area contained between the ordinates of the first and last instants of the time. And these areas, when taken from the beginning A , are as the squares of the times $A\ 1$, $A\ 2$, $A\ 3$, &c. or of the velocities $1\ d$, $2\ e$, $3\ f$, &c. by the property of similar figures. Therefore the sums of the velocities, and consequently the spaces described in any given terms of time taken from the beginning of an uniformly accelerated motion, are to each other as the squares of the times, or of the last acquired velocities. Hence M it likewise appears, that the spaces described in equal successive parts of time, are as the areas contained between A and $1\ d$, $1\ d$ and $2\ e$, $2\ e$ and $3\ f$, &c. which areas are to each other as the odd numbers 1 , 3 , 5 , 7 , 9 , &c. as appears by inspection from the number of equal and similar small triangles contained in each.

Again, suppose the motion at the end of the time N AB to become uniform with the last acquired velocity BC . Complete the parallelogram $BD\ C$, making BD equal to AB , and the ordinates $1\ i$, $2\ k$, $3\ l$, &c. will denote velocities, and consequently spaces described. Their sum will express the whole space described in a time equal to AB , and will be denoted by the area $BD\ EC$. But this area is equal to twice the area $AB\ C$. Consequently the space described O by an uniform motion with the last acquired velocity, during a time equal to that of the acceleration from the beginning, will be double the space described by the accelerated motion.

It

P It may readily be apprehended that an uniformly retarded motion is exactly the reverse of a motion uniformly accelerated. For suppose a constant force acting against a body in motion; as for example, gravity acting against the motion of a body projected directly upwards, it will destroy an equal part of the initial velocity in an equal particle of time. Now, if these equal deductions be called unities, and be successively taken from any number whatsoever, till the last remainder be nothing, it is evident that the series of remainders will be the natural numbers 1, 2, 3, &c. in a reversed order, and every thing that was proved of the times, spaces, and velocities (28, F. 30, K), or of the parts of the triangle ABC, fig. 5, will be true, *mutatis mutandis*, that is to say,

Q In the same body in motion, and retarded by a constant and equally acting force, the spaces described in coming to rest, are as the squares of the initial velocities (29, G. 31, L), or as, the squares of the times during which they are described: and are equal to half the spaces, which in an equal time would have been described by their respective initial velocities uniformly continued.

IMPULSE. ATTRACTION.

C H A P. V.

OF THE PRODUCTION, COMMUNICATION, OR DESTRUCTION OF MOTION.

MOTION is produced, destroyed, or changed **T** in a body, either by the impulse, collision or stroke of another body in motion, or by the force of attraction. Repulsion being immediately the contrary to attraction, and not being perhaps sufficiently general and universal (6) to be admitted as a common property of bodies, need not be here considered.

We do not know whether the distinction between impulse and attraction be real, and existing in the nature of things, or only relative to the imperfect state of our knowledge. The most observable difference is, that impulse is a force which acts from place to place, or, in other words, cannot be without motion: but attraction can exert itself even though no motion is produced.

To exemplify this, suppose two bodies to meet **x** directly with equal quantities of motion; the effect of the stroke will be, that the whole motion will be destroyed, and the bodies will remain together. The forces will likewise be destroyed, and the bodies may be moved apart; each with the same facility as if the other did not exist. In this case, we have supposed no attraction to be exerted by them on each other. But let it now be supposed, that their motion, instead of being uniform, and the consequence of their inertia, is produced by a mutual attraction. They come together, and the motion is destroyed as before.

fore. But the force of attraction, by which they were originally put in motion, remains, and is exerted in pressing them against each other.

y The smallest finite impulse can overcome the greatest finite pressure. For, let any pressure be supposed to produce acceleration, and the body, when in motion, will have more force than when it was merely pressed. In its acceleration from rest, it must pass through every possible velocity less than the velocity last acquired. Let the impelling body have a momentum expressed by the product of its mass into its velocity (19, 1). Whatever product this may be, it is possible to assume a period of the acceleration of the body pressed, in which its velocity shall be so small, as that its product into its mass shall be still less. And it has already been said, that the force of mere pressure is yet less than this. Consequently, it is less than that of the impulse.

z From this it is inferred, if two bodies, perfectly hard, or unyielding, were to strike each other with any velocity, that they would be broken to pieces, provided the cohesion of their parts were less than infinite.

A But if the cohesion were infinite, it is presumed that the communication or destruction of motion would

B be instantaneous. However, there are no such bodies found in nature, and very considerable difficulties arise in the abstract reasoning concerning them.

c There appears to be the same relation between pressure and momentum as between a line and a surface.

surface. In some respects, the first may be said to generate the latter. Both are capable of increase or diminution, and yet no increase or diminution of the one can produce the other.

That pressure, which gravity causes bodies to exert against any obstacle interposed between them and the earth, is called Weight. We have already taken notice of its effect when free to produce an uniform acceleration in falling bodies (26, A B C). If its power could be increased or diminished, it would proportionally increase or diminish the momentaneous velocities and spaces, (28, E) and consequently the whole space passed through in a given time; that is to say, constant forces are as the spaces x passed through by acceleration in a given time, or as the last acquired velocities.

When the effect of any retarding force is considered (31, F), the force will, in a given time, be as the whole space described during the retardation, till the motion is destroyed, or as the initial velocity.

Let two accelerating forces be to each other in any ratio, the last acquired velocities will (35, E) be in the same ratio. Imagine the less motion to be continued till its last acquired velocity becomes equal to that of the other, and the whole time (27, D) will then be to the former time as the greater velocity to the less, or inversely as the forces; that is, the times required to produce equal velocities G are inversely as the accelerating forces. But the spaces described in equal times are as the forces (35, E). Whence the spaces described in any other

36 ACCELERATING AND RETARDING FORCES.

- times will be as the forces and the squares of the times (29, G) jointly. But when equal velocities are produced, the squares of the times will be inverfely, as the squares of the forces (35, G). Therefore the fpaces in this cafe will be as the forces directly, and the fquares of the forces inverfely, or
- H inverfely as the forces; that is, the fpaces paffed through in producing equal final velocities are inverfely as the accelerating forces.
- I When the effect of a retarding force is confidered (31, P) the fpaces paffed through in deftroying motion are inverfely as the retarding forces, when the initial velocities are equal.
- K If a body be acted upon by a conftant and invariable force, and its mafs be either increafed or diminifhed, without altering the force, the effect will be the fame with refpect to acceleration, or retardation, as if the force, without changing the body, were diminifhed or increafed in the inverted ratio of the mafs. For the force being fupposed invariable, will always produce or deftroy the fame quantity of motion (21, Q) in a given time. This quantity will be meafured by the product of the mafs into the velocity (19, L). And that this product may continue unaltered, it is neceffary that the velocity fhould diminifh in the fame proportion as the mafs is increafed, and the contrary.
- L A body impinging with different velocities on tallow, clay, timber, and fome other fubftances, penetrates to depths in the fame fubftance which are as the fquares of the initial velocities. Whence it follows,

follows, (32, Q) that these substances oppose a constant and invariable force of retardation against the motions of given bodies.

Suppose a body to impinge on an uniformly resisting substance, if the initial velocity vary only, it will penetrate to depths which are as the squares of the velocities (32, Q). But if the mass (not magnitude) vary only, the consequence will be the same as if the retarding force had varied in the inverted ratio of the mass (36, K). And the depths or spaces will be inversely as the retarding forces (36, I) or directly as the masses: consequently, if both the mass and m velocity vary, the depths will be in the compound ratio of the masses and the squares of the velocities.

The dispute concerning the measure of forces, N which divided the philosophical world for considerably more than half a century, was founded on a partial consideration of the effects of collision. The O question agitated was, whether the forces of bodies in motion ought to be measured by the mass of matter multiplied by the velocity, or by the mass multiplied by the square of the velocity. The former affirmation was called the old opinion, and the latter the new opinion.

Neither of these opinions are sufficiently general P to apply to every case of motion, neither are they repugnant to each other, as the contenders for each insisted. The chief argument urged by the main- Q tainers of the new opinion was, that spheres of equal magnitude, but of different weights, being let fall into tallow, from heights that were inversely as the

D 3

weights,

weights, made pits of equal depths in the same. Now, said the disputants, equal causes are those which produce equal effects; the forces of these bodies at their impact on the tallow must be equal, as being the causes of equal effects, namely, the pits in the same substance. But the squares of the velocities of the impacts are (29, 0) as the heights from whence the bodies fall, or in this case inversely as the weights of the bodies. Therefore the product of each weight or mass into the square of its velocity is equal to the product of the other weight into the square of its velocity, when the pits, or, as it is affirmed, the forces, are equal.

- R All this is true, when it is considered as a mere explanation of the meaning of the word force, which, if understood and applied in this sense, will
 S not be productive of error. But when the above is intended to serve as a proof that the action of a body in motion cannot be measured by the mass multiplied into its velocity, it becomes necessary to observe, that the pit in the tallow being equal to another pit, does not prove that they were made by
 T equal powers or forces. For powers cannot be said to be equal, unless they produce equal effects in equal times; it being easy to imagine, that a weaker power continued for a longer time may produce an effect equal to that of a power of greater intensity,
 U though, of less duration. And it is evident, that these pits are not described in equal times; for they are equal to half the spaces which would have been described with their respective initial velocities uniformly

formly continued (32, s) in the respective times of description. But the initial velocities would describe such equal spaces in times which are inversely as the velocities themselves. And it has been already seen, that the result of the present experiment is an easy consequence of the properties of retarded motion, considered jointly with that definition which affirms the force or quantity of motion in a body to be as the product of the mass into the velocity (37, M).

The consideration that moving bodies penetrate w obstacles to depths which are as the mass of the body multiplied by the square of its velocity, is of great use in almost every circumstance of this kind. It follows from hence, that the depths to x which a body of given magnitude will penetrate into any substance, may be varied to infinity, without changing the quantity of motion. For the depths will always be greatest when the velocity is greatest (37, M); and the quantity of motion, or product of the mass into the velocity, will not be changed, if the mass be diminished proportionally while the velocity is augmented, and the contrary.

Thus it is shewn, that a small hammer, having r. the same striking surface and quantity of motion, will do more work at a blow than a large one. The driving of nails, or of piles into the earth, follows nearly the same law, though in the instance of the engine for driving piles by the fall of a weight, nothing (37, Q) would be gained by lessening

sening the weight and raising it higher: because, from the property of the mechanical engines, the heights to which weights can be raised in a given time are inverfely as their weights.

- z When a body in motion strikes another at rest, it does not communicate the utmost quantity of motion to this last, until its whole action has been exerted, which, if it penetrates, is not until it has either penetrated to its utmost depth, or passed through. Hence a projectile may pass through an obstacle without communicating any considerable quantity of its motion, provided the obstacle be considerably less thick than the depth to which the projectile must have penetrated by its whole effort.
- A This is exemplified in a pistol-bullet shot against a door set open, through which it passes without communicating motion enough to overcome the friction of the hinges: whereas a large piece of lead, having the same momentum, instead of penetrating, carries the door before it, even though the striking part be a prominence no larger than the bullet in the former case.
- B The different effects of motion, according as the velocity is greater or less, is shewn likewise in the vulgar experiment of breaking a stick, whose ends rest on two wine glasses. Fig. 6. Let the two wine-glasses A, C, be filled, and let the stick A C be placed with its ends resting on the two inner edges, as in the figure. Then, if a quick blow be struck downwards, on the middle part B, the stick will be broken asunder, without disturbing either the glasses

or their contents. In this experiment the two pieces AB , BC , are made to revolve on the points B and D , so that the points A and C rise up instead of pressing the edges of the glasses. On this account, if the blow at B be struck underneath instead of above the stick, the glasses will be broken.

That power or property by which a body recovers its figure, after it has been changed by any external action, is called elasticity.

If a body strike another at rest, and one or both of them yield inwards, or change their figure, the latter body will gradually, during the time of change, pass through every possible velocity from rest or nothing, to that which is expressed by dividing the quantity of motion in the striking body by the sum of the two bodies (19 , L 24 , U); at which last instant both bodies will have the same velocity, and will proceed uniformly together, provided neither be elastic.

But if both bodies be perfectly elastic, they will yield inwards, and the gradual change of velocity will obtain as before, till the instant of the utmost yielding, at which time both will have the same velocity. But the elasticity being supposed perfect, both bodies will recover their figure with a force equal to that by which it was changed. The action of the elastic force being contrary to that of the former stroke, will cause the two bodies to separate with the same relative velocity as they before came together. That is to say, the striking body will lose twice as much motion as it would have lost

lost by a similar collision without elasticity, and the body struck will in like manner gain twice as much. Consequently the striking body, after the collision, will either proceed forward, remain at rest, or be reflected back, according as its mass is greater, equal to, or less than that of the body struck.

- H The experimental methods of illustrating the theorems relating to motion have been made, for the most part, on pendulous bodies, or on bodies let fall, either along inclined planes, or in the open air. Mr. John Whitehurst, F.R.S. has contrived an instrument for measuring the time a body employs in falling through a given space. Its chief advantage consists in measuring, to an accuracy, considerably beyond the reach of the senses. The sense can with difficulty divide a second into twelve parts so as to reckon them: but this instrument divides it into one hundred. A hand or index, connected with wheel work, is made to revolve uniformly in a circle divided into one hundred equal parts, each revolution being performed in one second. The regulator of the motion is a fly, whose leaves may be set so as to displace a greater or less quantity of air, accordingly as the instrument is required to go slower or faster; or the same adjustment may be more accurately obtained by altering the weight that gives motion to the wheels. By the construction of the instrument the body is let go at the beginning of a revolution, and at the end of its fall it strikes another part of the mechanism, that instantly stops the hand, or index.

Mr.

Mr. G. Atwood, F.R.S. has invented an instrument for measuring the spaces passed through in a given time by a body in motion, whether that motion be accelerated, retarded, or uniform. It consists of two cylindrical boxes, suspended at the ends of a fine silk line that passes over a wheel or pulley. The axis of the pulley rests on the circumferences of four other wheels, so that the effect of friction is scarcely sensible. If the two boxes be equally loaded, the weight of the one will counterpoise or destroy the effect of the gravity of the other. The two boxes with their contents, together with part of the pulley, may, in this case, be considered as composing a mass void of gravity. Let any weight be added to one of the boxes, and as this weight is at liberty to move, it will be uniformly accelerated by the constant action of gravity (27, D): but by the construction of the instrument, it must give an equal velocity to the whole mass before mentioned, which, therefore, may be said to be an increase of its mass, while the moving force remains constant. The spaces passed through in a given time by the body will, therefore, be to the spaces it would have passed alone by the same action of gravity, (36, K. 35, E) as its own mass or weight is to the whole mass in motion. By this happy contrivance, the spaces passed through by acceleration or otherwise, in a given time, are rendered short, and easy to be observed with precision. For this intention there is a chronometer beating seconds added to the instrument, and a graduated rule near one of the boxes, with a moveable

moveable stage, to limit or terminate the motion*.

C H A P. VI.

OF THE ATTRACTION OF COHESION, AND OF SPECIFIC ATTRACTIONS.

- L** THE attraction of cohesion is that force by which bodies or their particles adhere to each other.
- M** It is demonstrated, that if the forces by which the particles of bodies tend towards each other decrease in the proportion of the squares of their distances, the attractive force of two spheres composed of such particles, will be governed by the same law; relation being had to the distances of their centers: and consequently, it will not be sensibly greater when they are in contact, than when they are at a small distance from each other. But
- if the first mentioned forces decrease in the proportion of the cubes of their distances, or in any greater proportion, the latter will decrease after a much higher rate, and the bodies, when in con-

* This instrument, and its various uses, are described at large by the inventor, in "A Treatise on the rectilinear motion and rotation of bodies."

taft, will attract each other very much more forcibly than when separated at the leaft diftance from each other*.

The firft of thefe attractions is gravity, as is oevinced by its action on the planetary bodies, and the latter appears to be the attraction of cohesion, r for its force is vastly lefs at the leaft diftance, than at the place of contact.

In confequence of this law, feveral deductions are made, which are found to agree with the phenomena of this latter kind of attraction, as,

Those particles which are poffeffed of large fur- q faces of contact, adhere moft ftrongly together, and form bodies which are called hard.

Those particles which touch each other in few r points, compofe bodies which are foft or fluid, on account of the fmall force with which their parts adhere together.

And hence probably the elasticity of fome bodies s may be explained; for it feems to depend on the cohesive force which reftores the particles to their firft relative fituation, when by any external impulse, they have been removed to a very fmall diftance from each other.

Many discoveries remain for the induftry of fu- t ture philofophers to make concerning this very powerful agent in nature.

By this power the drops of all fluids affume a u round form, and polished plates of metal adhere

* Principia I. 76.—I. 86.

v together with a prodigious force. This last is exemplified by paring a small part from each of two leaden bullets, and pressing the surfaces together; in which case, with a surface of contact not exceeding the twentieth part of a square inch, it will frequently require the force of 100lb. to separate them.

w By this power also it is, that liquids rise into the substance of bread, sponge, and other porous bodies; and are sustained in open capillary tubes a considerable height above the level. This height is in the reciprocal proportion of the diameters of the tubes.

x Two plain glass plates, fig. 7. touching each other at the line AB , and separated at CD , by a small obstacle K , being placed in the vessel of water $EFGH$, the water rises between them to the line DIA , which is an hyperbola.

Let two plain glass plates $ABCD$, fig. 8, be lightly moistened with oil of oranges, and placed one upon the other, so as to touch at the line AB , being kept separate at CD , by the small obstacle L interposed. In this situation let them be placed in the horizontal box, $EFGH$, the part CD resting on its bottom, and the other part towards AB , resting on the upper end of the perpendicular screw IK , which is fixed in the box for the purpose of raising the plates to any desired angle of elevation. Then a drop of the above mentioned oil being applied in the opening CD , will be attracted by the two plates, and will proceed with an accelerated motion

tion towards AB , if the plates are kept in an horizontal position. But if the end AB be raised by means of the screw, to a considerable angle, the drop will remain suspended in its course somewhere between CD and AB , suppose at N , and if the elevation be encreased, it will return towards CD , its weight overpowering the attraction of the plates.

Now, since the weight of the drop continues unaltered, it will not be difficult to find its tendency to return, or that part of its weight which is exerted in the inclined descent. For the proportion between that part and the whole, is as the height of the plane to its length, as will hereafter be shewn. And since the two powers, namely, the attraction by which the drop tends upwards, and that part of its weight which is exerted in the contrary direction, are equal when it remains suspended, the measure or quantity of the one will express the measure or quantity of the other. By these means it is easy to determine the attractive force; which is found to increase in the reciprocal proportion of the squares of the distances of the middle of the drop, from the end where the plates are in contact. That is, simply in a reciprocal proportion, because the drop enlarges its surface as the space becomes narrower; and again, simply in a reciprocal proportion, because the attraction increases, the nearer the plates approach each other.

This cohesive attraction extends to an extremely small distance from bodies, and where its power

terminates, repulsion takes place, of which we shall subjoin a few instances.

- A All hard bodies require a considerable force to bring them into contact, as appears by compressing a convex lens and plane glass together, which exhibit different appearances at the very point of supposed contact, according to the different degrees of compression. This is likewise shewn from the passage of the electric matter through metallic chains. Of which more hereafter.
- B When it rains on the surface of a vessel of water, small drops may frequently be seen running in all directions, which do not mix with the rest of the water for several seconds.
- C Hence likewise it is, that a body specifically heavier than any fluid may be made to swim on its surface; for, if a quantity of the fluid be displaced by the repulsion equal in weight to the solid, it will not sink.
- D Dry needles or thin plates of metal swim on water, and form cavities of a curve-lined form, extending to a considerable distance from the body.
- E Let ACB , fig. 9. represent the section of a vessel of water, on whose surface AB is laid two circular plates of tinfoil, on each of which is placed a small curtain ring, or some such body, to encrease its weight, and cause it to sink farther beneath the surface. By this means they may form two cavities about one-tenth of an inch deep, and extending half an inch every way from the circumferences of the plates. If they are brought within the distance of an inch from each other, they will rush together with an accelerated motion.

Things

Things remaining as in the last experiment, let *F* *D* and *E*, fig. 10, be two pieces of wet cork of the same dimensions; the water then, by adhering to their sides, may form a curve lined protuberance, extending about half an inch from their circumferences, and when they are brought within an inch of each other, they will rush together as before.

These appearances are easily accounted for, by *G* considering, that action and reaction are equal. The plate of tinfoil, by its repulsion, acts on the water, and prevents its filling the cavity, and the water by its weight reacts on the plate; but as this reaction is the same on all sides, no motion is produced. But when the two plates approach each other near enough to unite their repulsive actions, the weight of the water between them being diminished by the depression, its reaction is less than that which prevails on the opposite parts of the circumferences: consequently they move in the direction of the greater pressure, that is, towards each other.

In the latter case, the reaction is in a contrary *H* direction, being opposed to an attractive, instead of a repulsive force: for the reaction of the water endeavouring on all sides to return to its level, produces no motion, because its height is the same at equal distances all round. But when the two attractive powers are, by bringing the bodies nearer, made to act on the same water between them, it is raised higher above the common surface, and consequently reacts more strongly on that side; whence the bodies

Vgl. I.

E

rush

rush together, as in the former case. A depression of the surface between the two corks will, by diminishing the quantity of water, occasion them to recede from each other.

I It is not yet decided whether the attraction of cohesion, or the power by which bodies retain the aggregation of their parts, be one and the same with the attraction of combination, or chemical affinity. But as far as experiment has yet extended on this subject, there is reason to believe that they are the effects of different powers.

K When the rational method of philosophizing from observation and experiment was less known and esteemed than it is at present, many objections were made to the admitting attraction as a general cause. Among others, it was said to be a revival of the trifling philosophy of occult causes. But nothing can be more inconsistent and absurd than to compare that philosophy, which deduces general laws from the observation of phenomena, with the compendious method that vanity has invented to disguise or conceal human ignorance, by referring particular facts to occult causes. The laws of motion, the extension, the inertia, the resistance, and the attraction of matter, are deduced in the same manner; and if the causes of these be occult or unknown, it is not that philosophers are unwilling, but because they have not yet been able to discover them, or because some of them are, perhaps too, so simple, as to be referred to any other causes which our senses are adapted

to discern. Nothing, however, can be clearer, than that the existence of an agent or cause may be known, though it may itself arise from some prior cause of which we are ignorant; and it is enough that attraction really exists and acts according to established laws, to justify philosophers in admitting it in their explication of natural appearances.

B O O K . I.

S E C T. II.

Of Bodies in Motion.

C H A P. I.

OF THE MECHANICAL POWERS.

WHEN two heavy bodies or weights are made to act by any contrivance to act against each other, so as mutually to prevent each other from being put into motion by gravity, they are said to be in equilibrio. The same expression is used with respect to other forces, which mutually prevent each other from producing motion.

Any force may be compared with gravity, considered as a standard. Weight is the action of gravity on a given mass (35, D). Whatever therefore is proved concerning the weights of bodies will be true in like circumstances of other forces.

Weights are supposed to act in lines of direction parallel to each other. In fact, these lines are directed to the center of the earth, but the angle formed between any two of them within the space occupied by a mechanical engine is so small, that the largest

and most accurate astronomical instruments are scarcely capable of exhibiting it.

- o The simplest of those instruments, by means of which weights or forces are made to act in opposition to each other, are usually termed Mechanical Powers. Their names are, the Lever, the Axis and Wheel, the Pulley or Tackle, the inclined Plane, the Wedge, and the Screw.
- P In the theoretical consideration of these simple instruments, the parts they are composed of are imagined or supposed to possess no other properties than those which conduce to the purpose of their construction. Thus, they are all supposed to be without weight or inertia, and to move without friction. Many of these parts are taken to be mathematical lines, some perfectly inflexible, and others perfectly flexible, representing ropes. And these suppositions are allowable, because they imply nothing more than that the reasonings relate only to abstract notions or perfect instruments; and it is consequently no more to be urged that there are no perfect instruments, than that there are no perfect mathematical figures, at least that sense is able to discover or distinguish. For the difference between theory and practice is in some cases inconsiderable, and may in general be allowed for without much difficulty, from the general principles of mechanics, when once established.

C H A P. II.

OF THE LEVER.

THE lever is defined to be a moveable and inflexible line, acted upon by three forces, the middle one of which is contrary in direction to the other two.

One of these forces is usually produced by the reaction of a fixed body, called the fulcrum.

Let ac (fig. 11) represent an horizontal lever at rest. At the point b , equidistant from a and c , is placed the fulcrum d , and at the extremities a and c are hung the equal weights e and f . Then the lever will continue at rest, the weights e and f being in equilibrio. For it is evident, that if the line ac be moved on the fulcrum b , its extremities a and c will each be carried with equal velocities in the periphery of the same circle. And because ac is horizontal, the actions of the weights ef will be in direction at right angles to its length: that is to say, they will act in the direction of tangents to the said circle at the points a and c ; or they will act in the direction of those particles of the periphery, which may be imagined to coincide with the tangents. Each pressure therefore tends to move the correspondent extremity of the line ac in that very line of direction in which only it can move. Suppose the pressure at c to be removed, and the whole pressure at a will be employed in depressing the point a , or, which is necessarily in this case the

same, in raising the point c : on account therefore of the equal velocities of the points A and c , the action of E at the point A will be the same as if it were exerted at c in the direction of the tangent cG . But again, suppose the equal weight F to be restored, and the point c will then be acted on by two equal and opposite forces, which, destroying each other's effect, will not produce motion; consequently the lever will continue at rest.

T It is likewise evident, that if the radii AB and BC be not in a right line, the equal forces will nevertheless be in equilibrio, if they be applied in the directions of the tangents: thus, if BC be bent to the position BK , and the force F be there applied in the direction KH , the equilibrium will remain as before.

U If two contrary forces be applied to a lever at unequal distances from the fulcrum, they will equiperponderate when the forces are to each other in the reciprocal proportion of their distances. For,

V Let AC (fig. 12) represent a lever, whose radius AB is three times as long as BC . At A is suspended the weight E of one pound, and at c is suspended the weight F of three pounds. Then, I say, these weights will equiperponderate. With the radius BA describe the arc AK , intersecting CF at K . Draw the line BK , which may represent another arm to the lever; and it will be evidently of no consequence, whether the thread CF be fastened at c or K ; conceive it therefore to be fastened at K , and to act on the arm BK . Let AQ represent the force of E , and KR , being three times as long, will represent

represent the force of F . This force KF may be resolved (23, T) into two others, KI in the direction of BK , and KH in the direction of the tangent, and their quantities are determined by drawing the lines FH and FI , parallel respectively to KI and KH . Now, KI has no effect in moving the arm BK . It is the force KH alone that tends to produce motion towards H . The triangles BCK and KHF are similar, therefore, $BK : BC :: KF : KH$. But $BK : BC$, as 3 to 1, whence the force KH is $\frac{1}{3}$ of KF , as is likewise AG by the condition. Consequently HK and AG are equal, and being applied at the end of equal arms AB , BK , will be in equilibrio (56, T), which was to be proved; and the conclusion will be the same, when the weights are to each other in any other ratio, provided the arms of the lever AB and BC be reciprocally in the same proportion.

By the resolution of force it appears, that if two w contrary forces be applied to a strait lever at distances from the fulcrum in the reciprocal proportion of their quantities, and in directions always parallel to each other; the lever will remain at rest in any position.

For, let the forces, AE , CF (fig. 13) be resolved: x AE into GE parallel, and GA perpendicular to AC ; and CF into HF parallel, and CH perpendicular to AC ; and the forces, which tend to produce motion, will in all positions be to each other in the ratio of the forces applied; i. e. $AE : CF :: AG : CH$, the triangles AGE and CHF being similar.

Many

- y** Many curious and useful effects may be produced by levers, whose arms are bent into an angle; but the limits of this work do not permit us to enlarge upon them.
- z** It is evident, that all which has been said concerning the lever is equally true, when the contrary forces are applied on the same side of the fulcrum.
- A** On the lever *AB*, (fig. 14) if the weight *E* of one pound be applied at *A*, and the weight *F* of three pounds at *C*, so that their distances *AB* and *CB*, from the fulcrum *B*, may be as three to one, they will equiponderate; which is proved by applying the reasoning at fig. 12 to the present figure.
- B** Since, of the three forces which act on a lever, the two which are applied at the extremes are always in a contrary direction to that which is applied in the space between them; this last force will sustain the effects of the other two: or in other words, if the fulcrum be placed between the weights, it will be acted upon by, or will sustain their sum; but if the weights are on the same side of the fulcrum, it will be acted upon by their difference.
- C** On the principle of the lever are made, scales for weighing different quantities of various kinds of substances; the steelyard, which answers the same purpose by a single weight, removed to different distances from the fulcrum on a graduated arm, according as the body to be weighed is more or less in quantity; and the bent lever balance, which, by the revolution of a fixed weight, increasing in power as it ascends in the arc of a circle,

indicates the weight of the counterpoise. $A B C$ (fig. 15) is a bent lever, supported on its axis or fulcrum B in the pillar $J H$. At A is suspended the scale E , and at c is affixed a weight: draw the horizontal line $K G$ through the fulcrum, on which, from A and c , let fall the perpendiculars $A K$ and $c D$; then if $E K$ and $B D$ are reciprocally in proportion to the weights at A and c , they will be in equilibrio, but if not, the weight c will move along the arc $F G$, till that ratio is obtained. It is easy to graduate the plate $F G$ so as to express the weight in E by the position of c .

The beam of the common balance is usually a bent E lever, with equal arms. Its property of coming to rest in an horizontal position, when the extremes are equally loaded, is a consequence of its being bent, or, which is the same thing, of its fulcrum being above the line, joining the two points on which the scales are suspended. For it is evident, (fig. 16) that there is but one position in which the lengths of the arms $A B$, $B C$, referred to the horizontal line $D E$, can be equal, and that is when the points A and c are on the same level.

Balances that move with very little friction on the fulcrum, and are exactly equibrachial, are highly valued. But this last property is of less importance than is commonly imagined. For, if two balances be equally sensible, and one of them not equibrachial, it is certain, that if the standard weight be placed in one of the scales of this last, and counterpoised, and the standard weight be afterwards removed,

moved, any other body substituted in its place will have exactly the same mass, if it be in equilibrio with the counterpoise. In fact, this is the best method of weighing, when great accuracy is required. Or if the weights be always put into one scale and the quantities into the other, these last will be proportionally true among each other, which is quite sufficient in all philosophical experiments.

- G On this principle also depends the motions of animals, the overturning or lifting great weights by means of iron levers called crows, the action of nutcrackers, pincers, and many other instruments of the same nature.

C H A P. III.

OF THE AXIS AND WHEEL, AND OF THE PULLEY OR TACKLE.

- H THE axis and wheel may be considered as a lever, one of the forces being applied at the circumference of the axis, and the other at the circumference of the wheel, the central line of the axis being as it were the fulcrum. Fig. 17 is a perspective view of the instrument, and fig. 18 is a section of the same at right angles to the axis. Then, if AB , the semidiameter of the axis, be to BC , the semidiameter of the wheel, reciprocally as the power E is to the power F , the first of which is applied in the direction of the tangent of the axis, and the other in the direction of the tangent of the wheel, they will be in equilibrio
(36, v).

(56, u). For $A C$ may be conceived to be a lever, whose fulcrum is B , and whose forces applied at A and C are in the reciprocal proportion of their distances.

To this power may be referred the capstan or crane, by which weights are raised, the winch and barrel, for drawing water out of wells, and numberless other machines on the same principle.

The pulley is likewise explained on the principle of the lever. The line $A C$ (fig. 19) may be conceived to be a lever, whose arms $A B$ and $B C$ are equidistant from the fulcrum B . Consequently the two equal powers E and F , applied in the directions of the tangents to the circle in which the extremities are moveable, will be in equilibrio (55, s). And the fulcrum B will sustain both forces (58, B).

But in fig. 20 the fulcrum is at C , therefore a given force at E will sustain in equilibrio a double force at F , for in that proportion reciprocally are their distances from the fulcrum (56, u; 58, z). Whence it appears, that considering E as a force, M and F as a weight to be raised, no increase of power is gained when the pulley is fixed, as in fig. 19; but that a double increase of power is gained when the pulley moves with the weight (fig. 20).

A combination of pulleys is called a tackle, and N a box containing one or more pulleys, is called a block.

$A D B$ (fig. 21) is a tackle composed of four pulleys; two of which are in the fixed block A , and the

the other two in the block B that moves with the weight F. Now, because the rope is equally stretched throughout, each lower pulley will be acted upon by an equal part of the weight: and, because in each pulley that moves with the weight a double increase of power is gained; the force by which F may be sustained will be equal to half the weight divided by the number of lower pulleys. That is, as twice the number of lower pulleys is to 1, so is the weight suspended to the suspending force.

Q But if the extremity c (fig. 22) be affixed to the lower block, it will sustain half as much as a pulley; consequently the analogy will then be, as twice the number of lower pulleys, more 1 is to 1, so is the weight suspended to the suspending force.

It is for the most part more convenient to form tackles with blocks of the form exhibited in fig. 23.

R This reasoning depends on the equal tension of the rope, and is therefore conclusive only when the tackle is wrought by a single rope. In the system of pulleys (fig. 24) the power increases in a geometrical series, whose common ratio is 2, and number of terms equal to the number of pulleys. Thus, if a force be applied at A, it will be acted upon by half the weight F; if at B, by $\frac{1}{4}$; if at C, by $\frac{1}{8}$; and if at D, by $\frac{1}{16}$ &c. The reason of which is evident from what has been already said.

S It is evident, that in the composition of forces, the force produced is less than the sum of the compounding forces; A D (fig. 3) being always less than

than the sum of $A C$ and $A B$. On the contrary, in the resolution of force, a gain of force is produced, which is exemplified in the following instance:

The rope $E A C D F$ (fig. 25) is passed over the T pulleys A and B , and under the pulley C . Equal weights are suspended at E and F , whose actions on C may be represented by the equal lines $C J$ and $C H$. These forces compounded give $C D$, which will express the force exerted by the two weights E and F , tending to move C in the direction of the perpendicular, and is less than the sum of $C J$ and $C H$. Consequently a weight G being applied at C , whose quantity is less than the whole quantity of E and F in the same proportion, will sustain their effects, and remain in equilibrio. Therefore, if we consider E and F , as producing by composition a force equal to G , a loss of force ensues; and on the other hand, if G be considered as producing forces by resolution equal to E and F , an increase of force is acquired.

The quantity of this increase or diminution is u readily determined thus.

From J let fall the perpendicular $J K$ upon $C D$, then $C K$ will be the half of $C D$. And $J C$ is half the sum of $J C$ and $C H$. Now, as the whole of that sum is to $C D$, so is the sum of the weights E and F to the weight G (for they respectively represent the forces of those weights) and so is $J C$ to $C K$. But $J C$ is the secant of the angle formed between the rope $A C$ and the perpendicular $C D$, the line $C K$ being radius. Therefore, as the secant

cant of the angle formed between one of the ropes and the perpendicular is to radius, so is the sum of the weights E and F to the weight G .

- W** Hence it follows, that the general deduction, concerning pulleys and weights are only true when the ropes are parallel.

The pulley or tackle is of such general utility, that it is needless to point out any particular instance.

C H A P. IV.

OF THE INCLINED PLANE, AND OF THE WEDGE.

- W** THE inclined plane has in its effects a near analogy to the lever. Let AB be an horizontal plane on which the weight E is placed, and let ED represent the force exerted by the weight. AB may also be conceived to act as the arm of a lever, whose fulcrum is A . Let this lever revolve on its fulcrum from B to C , then the weight E will be found at e , and will act on the plane AC with an oblique force ed , equal and parallel to ED . Resolve ed into eb perpendicular, and bd parallel to AC , and the force eb will be destroyed by the reaction of the plane. With the other force bd , the weight will proceed with
.x an accelerated motion towards A . Whence it may be observed, that the inclined plane, acting against e in the manner of a lever, destroys that force which is exerted in the direction of the tangent of its line of motion, and that the acting force in this instrument is that which in treating of the lever
 was

was rejected (56, v), as having no effect. The y force with which any weight on an inclined plane tends downwards in the direction of the plane, is to the weight itself, as $b d$ to $d e$. Or as $e f$ to $A e$, which is the ratio of the height of the plane to its length, because the triangles $b e d$ and $f e A$ are similar. But $e f$ is to $A e$ as the sine of the angle the inclined plane makes with the horizon is to radius. See fig. 26. Therefore, as the said sine z is to radius, so is the force tending downwards in the direction of the plane to the weight. And because radius is a constant quantity, the forces by which the same weight tends downwards in the directions of various planes will be as the sines of their inclinations.

This instrument is not much used in its simple form.

If it be required to shew what force in the direction $e p$ parallel to $A B$ (fig. 27) will sustain the weight e in equilibrio. Set off $e m$ equal to $b d$, which will represent its force or tendency in the direction of the plane, and equal, but on the contrary side, set off $e n$, which will represent the force that, applied in the opposite direction, will sustain the weight in equilibrio. Draw $n p$ perpendicular to $A c$ and $e p$ parallel to $A B$, intersecting $n p$ in p , $e p$ will be the force required; for it is composed of $e n$ and $n p$, and $n p$ being perpendicular to the direction of the tendency of e avails nothing. Join $p d$ and this last found force is to the whole weight of e , B as $p e$ to $e d$, or as $e f$ to $f A$, which is the ratio of

the perpendicular height of the plane to its horizontal base, for the triangles $p e d$ and $e f a$ are similar*. And since action and reaction are equal, and in contrary directions (22, R), it is evident that the same force $e p$, which sustains e on the fixed inclined plane $c a b$, applied in the contrary direction would, if the plane be supposed moveable in the direction of its base $a b$, and the body e fixed by the application of an obstacle $q r$, sustain the effort with which the said body tends to impel the plane from e towards p .

D The wedge is composed of two inclined planes joined together at their common base, in the direction of which the power is impressed.

E Let $A B C$ (fig. 28) represent a wedge, whose vertex A is inserted between the two bodies D and E , which being fixed in position, resist in a certain degree any force which tends to separate them. This resistance usually is, like the weight in the inclined plane, perpendicular to the base $A F$, and the power, or force employed to overcome it, is

* The similarity of these triangles not being obviously deducible is proved thus:

Prolong $p e$ and $d m$ till they meet in s ; and the right angled triangle $m s e$ will be equal and similar to the triangle $n p e$, $s e$ being equal to $e p$.

The triangles $s e d$, $p e d$ have the two sides $s e$, $e p$ equal, the side $e d$ common to both, and the included angles $s e d$, $p e d$ are both right angles. Consequently the triangles are equal and alike in all respects. Euclid I. 4.

But it is easily shewn, that the triangle $s e d$ is similar to the triangle $e f a$, and so likewise must $p e d$.

impressed

impressed as was just mentioned, in the direction of the said base. Therefore, by the property of the inclined plane, the force required to keep one half $c f a$ of the wedge in equilibrio with the pressure of the body d , is to that pressure as $c f$ to $f a$. But as the pressure on the other half of the wedge acts with equal effect, a double force will be required to preserve the equilibrium, that is, a force as $c b$ to $f a$. Or, in general terms, in any f wedge, as the line $c b$, joining the two equal sides $a b$ and $a c$, is to the distance between the vertex a , and the middle point f of $c b$, so is the force impressed to the resistance in d and e .

This instrument is commonly used in cleaving α wood, and was formerly applied in engines for stamping watch-plates. The force impressed is commonly a blow, which is found to be much more effectual than a weight or pressure. This difference is usually accounted for, by supposing that the tremulous motion produced by the stroke, considerably diminishes the very great friction at the sides. But there is no doubt, that it is chiefly referable to the principles that obtain when resisting bodies are penetrated (37, m).

All cutting instruments may be referred to the π wedge. A chisel, or an axe, is a simple wedge. A saw is a number of chisels fixed in a line. A knife may be considered as a wedge when employed in splitting, but if attention be paid to the edge, it is found to be a fine saw, as is evident from the much greater effect all knives produce by

a drawing stroke, than what would have followed from a direct action of the edge.

C H A P. V.

OF THE SCREW, AND OF MECHANICAL ENGINES IN GENERAL.

- I** THE Screw is composed of two parts, one of which is called the screw, and consists of a spiral protuberance, called the thread, which is wound or wrapt round a cylinder; and the other, called the nut, is perforated to the dimensions of the cylinder, and in the internal cavity is cut a spiral groove adapted to receive the thread.
- K** Let $A D G E$ (fig. 29) represent a cylinder, and $A B C$ any flexible substance of a thickness altogether inconsiderable or evanescent. Suppose $A B C$ to be a triangle, having a right angle at A , and one of the legs $A B$ containing the right angle to be applied to the cylinder in a line parallel to its axis. Imagine now the cylinder to turn on its axis so that the triangle $A B C$ may be rolled or wrapped close on its surface. The lines $B C$, and all others, as $I K$, $L R$ parallel to it, will then be contiguous to, or coincident with, the peripheries of circles, whose planes are all at right angles to the axis, and consequently parallel to each other. But the line $A C$ will become a curve $A Q L M A N O P$, &c.
- L** which is called an helix. This curve will always, or in every part, proceed from one towards the other

other end of the cylinder it enwraps, and will make equal angles with the generating circle of the cylinder. For any one of these angles, $\angle K I$, will be produced by the application of another angle $\angle K I$, always equal to the angle $\angle A C B$.

Suppose the cylinder $A D O E$ to be perpendicular to the horizon, the lines $B C$ and its parallels, together with all their correspondent circles on the cylinder, will then become horizontal. Let the line $A C$ now represent an inclined plane whose height is $A B$, and the helix being of the same length and height, and equally inclined to the horizon throughout, will not differ in mechanic effect from the inclined plane. That is to say, the tendency of a weight to descend on the inclined plane, will be exactly the same as on the helix.

Let $A L$ be the perpendicular distance between two adjacent threads. Draw the horizontal line $L R$ intersecting $A C$ in R . Then $L R$ will be equal to the circumference of the cylinder, and $A R$ will be equal to one revolution of the helix. But $A R$ represents an inclined plane, equivalent in power to the helix. Every helix therefore is equivalent in power to an inclined plane, whose length is equal to one revolution of the helix, and height equal to the distance between two adjacent threads measured by a line parallel to the axis of the cylinder contained within the helix.

If the horizontal thickness of the nut be disregarded, it will not differ from a weight to be sustained on the helical plane. Consequently it

will be kept in equilibrio by an horizontal force, which is to that of the weight, as the perpendicular distance between two adjacent threads is to the circumference of the cylinder (69, P. 65, A B).

s Or if the power be applied in the direction of the threads of the screw, the equilibrio will be had when the power is to the weight as the perpendicular distance between the two adjacent threads is to the length of one thread of the screw (69, P. 65, x).

T But there are few, if any, instances where the screw is used without the lever. If an arm EF (fig. 30) be applied to move the nut, the weight and the power may be considered as acting upon a lever, whose fulcrum is at the axis of the cylinder.

v And, therefore, the proportions last found (R, s) must be compounded with the ratio of the semi-diameter of the cylinder to the distance of E from the axis of motion.

v It would be difficult to enumerate the very many uses the screw is applied to. It is extremely serviceable in compressing bodies together, as paper, &c. It is the principal organ in all stamping instruments for striking coins, or making impressions on paper or cards, and is of vast utility to the philosopher, by affording an easy method of measuring w or subdividing small spaces. A very ordinary screw will divide an inch into five thousand parts; but the fine hardened steel screws, that are applied to the limbs of astronomical instruments, will go much farther. This method will be readily understood from the contemplation of fig. 31.

On

On the rule *E G H F* is fixed the upright piece *E F*. *x*
 Through this piece the stem of the screw *A B* passes, and is held by a collar, so that it may be moved on its axis without advancing or retiring in the direction of its length. The circular plate *C D* is fixed at right angles to the axis of the screw which passes through its center. The piece *I K M L* is adapted to slide lengthways on the rule. This piece has a square aperture, across which is stretched a fine wire *O P* at right angles to the graduated line on the rule that may be seen beginning at *N*. At *I K*, on the sliding piece, is a raised part perforated helically to receive the screw. Suppose the screw to have fifty turns in the length of an inch, and the edge of the plate to be divided into 100 equal parts: suppose likewise, that the wire *O P* stands between two of the dividing lines, and that it is required to determine its distance from one of them. Turn the screw, which of course will move the sliding piece, and observe, with a magnifier, when the wire accurately covers the dividing line. Then the number of whole turns of the screw employed in the operation will give as many fiftieth parts of an inch, and the odd divisions of the plate *C D* will shew the number of hundredth parts of a turn, that is to say, hundredth parts of one-fiftieth of an inch, or five thousandth parts of an inch.

The screw has been applied with great success in *y* the division of astronomical instruments*.

* By Mr. Ramsden, who has written a treatise on the subject.

- z** It is easy to conceive, that when forces 'applied to mechanical instruments are in equilibrio, if the least addition be made to one of them, it will preponderate and overcome the effort of the other. But the want of a perfect polish or smoothness in the parts of all instruments, and the rigidity of all ropes, which increases with the tension, are great impediments to motion, and in compounded engines are found to diminish about one-fourth of the effect of the power.
- A** The properties of all the mechanical powers depending, as has been shewn, on the laws of motion laid down in the beginning of this treatise, and the action, or tendency to produce motion, of each of the two forces, being applied in directions contrary to each other, the following general rule for finding the proportion of the forces in equilibrio on any machine will require no proof.
- B** If two opposite forces be applied to the extremes of any mechanical engine, in the direction of the lines, in which, by the construction of the engine, the said extremes would move, and the intensities of the forces be to each other reciprocally as the velocities the extremes when put in motion would acquire in the same indefinitely small time: then those forces will be in equilibrio.
- Suppose the forces to be weights, and the same may be expressed thus.
- c** If two weights applied to the extremes of any mechanical engine be to each other in the reciprocal proportion of the velocities resolved into a perpendicular

pendicular direction, (rejecting the other part) which would be acquired by each when put in motion for the same indefinitely small time, they will be in equilibrio.

Whence it may be observed, that in all contrivances by which power is gained, a proportional loss is suffered in time. If one man, by means of a tackle, can raise as much weight as ten men could by their unassisted strength, he will be ten times as long about it.

It is convenience alone, and not any actual increase of force, which we obtain from mechanics. This may be illustrated by the following example:

Suppose a man at the top of a house draws up ten weights, one at a time, by a single rope, in ten minutes. Let him have a tackle of five lower pulleys, and he will draw up the whole ten at once with the same ease as he before raised up one; but in ten times the time, that is, in ten minutes. Thus we see the same work is performed in the same time, whether the tackle be used or not: but the convenience is, that if the whole ten weights be joined into one, they may be raised with the tackle, though it would be impossible to move them by the unassisted strength of one man.

Or, suppose, instead, of ten weights a man draws ten buckets of water from the hold of a ship in ten minutes, and that the ship being leaky, admits an equal quantity in the same time. It is proposed, that by means of a tackle, he shall raise a bucket ten times as capacious. With this assistance he performs

performs it, but in as long a time as he employed to draw the ten, and therefore is as far from gaining on the water in the latter case as in the former.

- H Since then, no real gain of force is acquired from mechanical contrivances, there is the greatest reason to conclude, that a perpetual motion is not to be obtained. For in all instruments the friction of their parts and other resistances continually destroy a part of the moving force, and at last put an end to the motion.

C H A P. VI.

OF THE CENTER OF GRAVITY.

- I LET A B (fig. 32) represent a long slender body of an inconsiderable thickness, which is attracted by another body in the direction of the small parallel arrows, a b c d, &c. Then the motion of A B will be the sum of the motions of all the parts situate between A and B. Interpose the pointed obstacle c, D, and A B may be considered as a lever; c being the fulcrum. Consequently, if c be so placed that the parts between A and c may be in quantity and distance from the fulcrum equipollent to those between c and B, the whole body will rest in equilibrio on the point c. This point is called the center of gravity.
- K The thickness of A B being inconsiderable, the point c may be esteemed as the center of gravity, but is not so when the thickness is taken into the account.

account. The foregoing illustration, besides the advantage of its simplicity, may serve to shew that when we speak of the whole attractive force of a body being collected in its center; as for example, the center of the earth, it is not to be imagined that any real power, or, as it were magic force, is supposed to exist in that center. In the same manner the body *A B* ceases to move, not immediately because its center of gravity is sustained, as if the cause of motion existed in that center alone, but because, by the property of the lever, the forces on the side *c B* are made to counteract and destroy those on the side *c A*.

The center of gravity is defined to be a point *L* about which all the parts of a body or bodies are in equilibrio.

Therefore, the center of gravity of two bodies, *A* and *B* (fig. 33) will be a point *c*, in the right line that joins their centers of gravity, which is distant from the center of each body in the reciprocal proportion of their masses (56, *v*); that is, $A c : c B :: B : A$. And the center of gravity of three bodies, suppose *A*, *B* and *E*, will be found at *D* in the line *c E*, which joins the center of gravity of *E* with the point *c*. *c D* being to *D E* reciprocally as the sum of the masses of *A* and *B* is to the mass of *E*. For it is easily proved, supposing the lines to be levers, that the bodies *A* and *B* will equilibrate on the point *c*, which, as the fulcrum, will sustain both their forces (58, *B*); and also, that the body *E* will equilibrate with the force sustained at *c*; *D* being the

the fulcrum: In this manner the center of gravity of any system of bodies may be found.

- N Though the point called the center of gravity is defined from the universal property, gravity, yet, it may be as well defined from the inertia of matter; for the point *c* (fig. 32) is the center of inertia, or the point at which an impulse will move the body without producing rotation; or, if the body were in motion in a right line, without rotation, the point *c* is the only point at which the opposition of a fixed obstacle will destroy the whole motion at once; but our intended conciseness forbids the elucidation of rotatory motions.
- O If two bodies move uniformly in right lines, their common center of gravity will either be at rest, or will move uniformly in a right line; and the same is likewise true of the center of gravity of three bodies, for the center *c* (fig. 33) of any two of them may be considered as one body. Therefore, if *c* and *e* be in motion, the common center *d* will either be at rest, or will move uniformly in a right line. And the same may thus be shewn of any number of bodies*.
- P The common center of gravity of two or more bodies does not change its state of motion or rest from the mutual actions of the bodies upon each other; and therefore, the common center of gravity of all bodies mutually acting upon each other, is either at rest, or moves uniformly in a right line,

* Principia, in Corol. 4. ad 3. Legem motus.

actions and impediments from without being excluded. For,

If the bodies *A* and *B* (fig. 33) act upon each other, the motion produced in each will be equal, (22, *R*) and the ratio of *C A* to *C B* will consequently remain the same, whether they approach to, or recede from each other. The state of *c* will not therefore be changed by their mutual actions. If the third body *E* be added to the system, the center *D*, for the same reason, will not be changed, as to its state of motion or rest, whether *E* acts upon *c* or not: and the same may be proved of any number of bodies.

Since then the state of the center of gravity of *R* any system of bodies, as to rest, or uniform direct motion, is not affected either by the motions or mutual actions of the bodies of which it is composed, external actions or impediments being excluded, it is plain that the same law holds good in the motion of a system of bodies as is observed by a single body. For the progressive motion of a single body, or of a system of bodies, must be estimated by the motion of the center of gravity.

Hence it is that the center of gravity of the *s* earth is not affected by the motions on its surface, or in its bowels. When a projectile, a cannon ball, for instance, is thrown upwards, the projecting force reacting on the earth, causes it to move in the contrary direction; but as the motions are equal, the center of gravity remains the same.

T The motions and actions of bodies upon each other in a space that is carried uniformly forward, are the same as if that space were at rest.

U For the motions and actions of bodies upon each other depend on their relative motion, the velocity of which is the sum of their absolute velocities, when they are moved in opposite directions, or their difference when they move in the same direction. And this sum or difference is not altered by an equal velocity impressed on all the bodies in the same or a parallel direction, as in the present case: since, when two bodies move in contrary directions, in a space carried uniformly forward, the velocity added to that body, with whose motion the * motion of the space conspires, is exactly equal to the velocity destroyed in the other body, whose motion is opposed by that of the space; and when the bodies move in the same direction, an equal velocity being added to, or destroyed in both, the difference is

V likewise unaltered. This is likewise confirmed by daily experience; motions performed on board a ship under sail are the same as if the ship were at anchor; except so far as they may be disturbed by the irregular tossing of the waves, which affects them successively, as much in one direction as another. A fleet of ships carried by an uniform

* Space being in its own nature immoveable, the expression is here improper; but it conveys a clear idea of the proposition in concise terms, though we can form no idea of bodies included in a space being acted upon by that space. The space here mentioned is merely ideal, may be called relative, and is defined to be a moveable dimension.

current,

current, either preserve the same relative positions, or approach to, or recede from, each other in the same manner as they would if no such current existed. And the motions of bodies at the surface of the earth are no otherwise affected by its revolution on its axis than as the revolution is not rectilinear, the effects of which, though considerable, are not enough so to fall under common observation.

This proposition is likewise true, if the motion w of the space be uniformly accelerated, or, which is the same thing, if all the bodies be constantly acted upon by parallel forces which act equally, according to their masses, on each of them.

For such forces will cause all the bodies to move x with the same acceleration, and to describe equal spaces in the same direction with each other. They will not therefore change their relative motions or situations.

C H A P. VII.

OF PENDULUMS.

Y THE bodies spoken of in the present chapter are supposed to move without rotation, friction, or resistance from the air, or any other medium; neither are the magnitudes of the bodies brought into consideration.

It has already been shewn (65, *y*), that the force of a body to descend along an inclined plane is to the whole force of its gravity as the height of the plane to its length. If the body be at liberty to descend on the plane, the first-mentioned force will, by its constant and equal action (27, *v*) produce an uniform acceleration.

The spaces described from the beginning on a given inclined plane are (29, *g*) as the squares of the times; that is to say, the times of description of inclined planes of the same inclination are as the square roots of their lengths.

B The final velocities (36, *h*) of accelerated motions being equal when the forces are inversely as the spaces passed through, and the length of an inclined plane being to its height in this same ratio of the forces, by which a body would descend along the plane, or fall freely through its height, it follows that the final velocity acquired by a body that descends along such a plane is equal to the final velocity it would acquire by falling freely through its height. Hence also the final velocity is always equal

equal when a body has fallen through an inclined plane of a given height, whatever may be its length.

The times of acquiring this given velocity, or of x passing over the whole lengths, will be inversely as the forces (35, α); that is to say, directly as the lengths when the heights are equal (65, γ).

Bodies that descend from a given height always acquire the same final velocity, whether they descend along a single plane or many. Let EG (fig. 34) represent an horizontal, and AG a perpendicular line; and suppose a body to descend along the inclined planes AB , BC , CD , DE ; continue AB to F in the line EG , and draw the lines BK , CI , DH , parallel to the horizon. The body after passing through AB and BC will have acquired a velocity equal to the velocity it would have acquired simply by descending along BC or (80, δ) along BL , added to the velocity it had at B : therefore the velocity at C , after passing through the planes AB , BC , is the same as would have been acquired by descending from the same height AI through a single plane AL . The same reasoning may be extended to prove, that when the body has arrived at D , it will have the same velocity as it would have acquired by descending in one plane AM of the same height AH : and so forth for any number of planes whatever.

Since the planes along which a body may pass, in descending from a given height, are not limited either in number or magnitude, we may assume them to be indefinitely small, and indefinitely numerous.

They may then be conceived to form a curve, and
 o it will follow, that the last acquired velocity of a body that descends by a gravity from a given height along a surface either plane, polygonical or curved, is always the same, and is equal to that velocity it would have acquired by descending from the same height by the action of its gravity in free space.

H On the planes of the same inclination the times of descent from the beginning are as the square roots of the lengths of the planes (80, z. 29, G).

I If a body descend along any number of inclined planes, $A B, B C, C D, D E$, (fig. 34) and another body descend along a like number of planes, $N O, O P, P Q, Q R$, (fig. 35) having respectively the same inclination and proportional lengths, namely, $N O$ to $A B$ as $O P$ to $B C$, and as $P Q$ to $C D$, &c. then the times of descent will follow the same law as would have obtained if each had passed down a single
 K plane of the same inclination: that is to say, they will be as the square roots of the lengths passed over. For, if the bodies were each to descend singly along any two correspondent planes $A B$ and $N O$, the times would be as the square roots of those lengths (82, H): and the final velocities would be in the same ratio (29, G). The same will be true of the planes $B C$ and $O P$. But now, suppose each body to have descended through two planes $A B, B C$, and $N O, O P$, then the time of descent through $B C$ will be less than it would have been in proportion as the velocity it has at B is greater, namely,
 in

in proportion to the square root of AB (29, G). And in like manner the time of passing singly through OP will be diminished in proportion to the increase of velocity gained at O , or in proportion to the square root of NO . But AB is to BC as NO to OP , and therefore the times of passing through BC and OP are diminished in proportion to their magnitudes, and must continue to have the same ratio as before. Again, since NO is to AB as OP to BC , the sum of NO and OP , namely, NP , will be to the sum of AB and BC , namely, AC , in the same ratio; and the sum of the square roots of NO and OP , namely, the square root of NP , will be to the sum of the square roots of AB , BC , namely, the square root of AC , as the square root of NO to the square root of AB , or as that of OP to that of BC . And since it has been shewn, that the times of passing over the planes NO , OP , and AB , BC , are respectively proportional to the square roots of their lengths, the whole times must be proportional to the square roots of their sums, or whole length, which was to be proved: and the same reasoning will apply to any number of planes.

If the planes be indefinitely short and numerous, L they may be conceived to form a curve, and a similar assemblage of planes respectively, in proportion to the former, will form a similar curve. The foregoing arguments will then prove, that bodies descending along similar curve surfaces describe them in times which are as the square roots of the lengths of the curves.

M If a body be urged towards a given point, by a force which is proportional to the distance of the body from the point, it will always arrive at the point in the same time, whatever the distance may be. Let I (fig. 36) be the point; and suppose two bodies to be let fall from any two points, A and R at a finite distance from I . Join AR , AI , and RI ; and imagine AI to be divided into an indefinitely great number of equal parts, which, consequently, will each be indefinitely small. Through each point of division in AI , imagine the lines BQ , CP , DO , &c. to be drawn parallel to AR , and they will divide the distance RI into an equal number of indefinitely small parts. Any two intervals, AB and RQ , will be in proportion to the whole lines AI , RI , of which they are like parts. Now, the force may be esteemed to be invariable or constant, while the body passes over the interval AB , because the distance is not definitely less, during that time: and the same may be said of the interval RQ ; that is to say, the forces being as the distances AI , RI , will be also as the intervals or spaces AB , RQ , passed through: and the times of passing those intervals will be equal (35, E). By the same argument the succeeding intervals BC , QP , would have been passed over in equal times, if the motions had commenced at B and Q . But in the present case, the velocities already acquired in B and Q are (35, E) proportional to the spaces AB and RQ , and consequently are such as would, without any other action, carry the bodies over the spaces BC and QP , in equal times.

It

It is therefore evident, that the bodies are carried over the second spaces bc and qp , by means of actions or forces which would have carried them respectively through those spaces in the same time, increased by acquired velocities that would likewise have carried them through the same in equal times. The acquired velocities in b and q must therefore subduct equally from the times in which bc and qp would have been otherwise described, and the remaining or actual times of description will be equal. This reasoning will extend to prove, that all the other correspondent intervals, cd and po , de and on , &c. are respectively passed over in equal times; or, in other words, the whole lines ai and ri will be passed over in equal times; which was to be shewn.

If a number of indefinitely short inclined planes n be joined together, and the sine of the inclination of any plane to the horizon (65, z) be as its distance from the lowest plane, measured along the planes, the tendency of a body to descend on them will be as its distance thus taken: consequently (84, m) a body will, by passing along them from any distance, arrive at the lowest point in the same time. The line in which the descent is made may be conceived to be a curve, because no part of any definite magnitude lies in the same right line. This curve is termed a cycloid.

Every thing that has been proved of the accelerated descent of bodies along inclined planes will hold good, mutatis mutandis (31, p) when bodies are

retarded in their ascent along the same, or congruous planes.

Q A pendulous body oscillates by the same laws as it would move on an inclined surface of the same figure as the curve it describes. For the reaction of the string or rod is exerted against, and destroys the very force that such a plane would destroy.

R The pendulums of clocks usually vibrate in the arcs of circles. It has formerly been thought an advantage to make them vibrate in the arcs of cycloids; but the difficulties that attend the practical application are such, that there is good reason to think that they produce greater errors in the admeasurement of time than those they are intended to remedy*. For this reason, we shall here only explain the properties of bodies vibrating in circular arcs.

S Let B (fig. 37) be a body pendulous from A, and moveable in the arc B F E, whose lowest point is F: draw the line A F C, on which from B let fall the perpendicular B D. This last line B D will be horizontal. From B draw B C at right angles to A B, and consequently touching the arc B F E. The body at B will then be urged towards F by its gravity in the same manner as if it rested on an inclined plane B C, making the angle D B C with the horizon. Which angle, by reason of the similarity of the triangles D B C, D A B, is equal to the angle D A B, or the in-

* The excellent Huyghens has explained the vibrations of pendulous bodies in his *Treatise de Horologio Oscillatorio*; and the same thing is performed more generally in the *Principia* I. § 10.

clination of the pendulum-rod or string with the perpendicular. But the force at B is as the sine of this angle (65, 2). Now, in very small angles, the sines, and consequently the forces, are nearly proportional to the subtending arcs to be passed over: therefore, (85, N) all the circular vibrations T of the same pendulum are nearly equal when the arcs of vibration are small.

But, in fact, because the arcs increase faster than u the sines, the less vibrations of the same pendulum are performed in less times.

The times of the vibrations of pendulums that v describe similar arcs of circles, or, which is in fact the same, have equal angles of vibration, are (83, 1.) as the square roots of the lengths of the arcs; and, because the similar arcs are as the radii they are described with, the times will be as the square roots w of the lengths of the pendulums. This proposition x may be affirmed likewise of all angles of vibration (87, T) when they are small.

Thus far, in our reasoning, we have supposed the y force of gravity to be invariable in a given body; but if it be supposed to vary, its effects will vary in the same ratio (35, D, E); and the lengths of similar arcs described by a pendulous body in equal times will be directly as the forces of gravitation that urge them. Now, the lengths of the arcs are as their radii, that is to say, the lengths of pendulums, vibrating through small arcs (87, x) and measuring equal times, are as the gravitating forces.

Z If the ball **B** (fig. 37) of a pendulum were let go from any point remote from the lowest point **F**, it would descend to **F** with an accelerated motion through the arc **B F**, and if its motion were without all resistance or impediment, it would by a retarded motion precisely similar to its former acceleration, ascend through an arc **F E**, equal to **B F**. From the point **E** it would again descend by acceleration to **F**, and again rise to **B** by passing through the arc **F B** in the same manner as it before ascended through the arc **F E**: the oscillation would thus continue for ever, the angles and the times of all the vibrations being equal. But this cannot be; for there is no avoiding a certain degree of friction at **A**, and the ball **B** striking, and giving motion to the parts of the air opposed to its course, must itself lose as much motion as it communicates, (22, R) by which means the motion must continually decay, and at last become insensible.

A The pendulums of clocks are maintained in their motion by the action of the wheels, which are driven round by means of a weight or spring. Let **E G F** (fig. 38) represent the swing-wheel of a clock that is urged to revolve in the direction **E G F**; let **c** and **d** represent the pallets moveable on an axis at **A**, and so connected with the pendulum **A B**, that they are made to vibrate along with it. Suppose the ball to vibrate from **R** to **B**, one of the teeth of the wheel resting against the pallet **c**, whose figure may be seen enlarged at **I K L**. When the pendulum returns

returns towards Q , the pallet c is drawn out, the wheel pressing first along the plane IK , and afterwards on the inclined plane KL ; by its action on which last, it pushes the pallet or assists its motion, till at length the tooth slips off the point L . During this time the pallet d , whose figure is seen at PNO , is carried in between the teeth on the other side of the wheel; that when the wheel escapes the pallet c , another tooth drops on the plane NP of the pallet d . The returning vibration again draws out the pallet d , the tooth of the wheel assisting its motion by pressing along the inclined plane NO till it escapes at O . At this instant the pallet c has acquired its original situation, and therefore receives the adjacent tooth: and the whole proceeds as before. This is a very simple and good escapement. It is called the dead-beat escapement, because the second hand in clocks of this construction falls with a single or vulgarly a dead stroke on the division line of the dial, and does not recoil, but remains motionless during that part of the vibration in which the tooth of the crown-wheel rests on the plane IK , or PN of the pallet.

There are many other escapements which our present purpose will not admit of describing. The leading requisite of a good escapement is that the impulse communicated to the pendulum, shall be invariable notwithstanding any irregularity or foulness in the train of wheels.

If the resistances arising from the friction at the
moving

moving parts, and from the motion communicated to the air, were always the same, and the clock were urged by a weight, the action of the swing-wheel on the pallets would be always the same at a given place, in consequence of which, the figure of all the parts being supposed invariable, the arc of vibration would be constantly of the same magnitude, namely, such as, that the motion lost by the resistances opposed to the pendulum should be accurately equal to the motion communicated by the pallets, and the times would be equal; that is to say, the clock would be perfect, and would measure time accurately. But these conditions are not easily obtained. It is not found, that the variation in the resistance of the air, arising from its changes of density, occasions any sensible irregularity in clocks. The most considerable irregularities in the movement arise from the tenacity of the oil applied to the moving parts. For the oil is less fluid in cold than in hot weather; and when it is less fluid, a greater quantity of the maintaining power must be lost in overcoming its rigidity: whence it must happen, that the teeth of the crown-wheel will in that case act less forcibly on the pallets, and the vibration will be less. If the pendulum be suspended on an axis, this cause, together with the constant wear, is very noxious; but this defect is remedied by suspending it by a stait flexible spring. And in addition to this, there have not yet been found any materials

which do not expand by heat. The pendulum rod therefore is longer in hot than cold weather, and the clock consequently goes slower (87, w).

It has not been determined from experiment and observation, how far these causes respectively affect the regularity of the clock's going. There are many good inventions for obviating their effects.

The irregular action of the maintaining power, in consequence of its giving motion to the whole train of wheels, is rendered of no consequence by means of the escapements, which are called detached or free escapements. For in these the impetus of the swing-wheel is not suffered to act on the pendulum, but is employed in raising a small weight in each vibration, which, by its fall, always gives the same impulse to the pendulum.

The expansion or contraction of deal-wood lengthways, by change of temperature, is so small, that it is found to make very good pendulum-rods. The wood called sapadillo is said to be still better. There is reason to believe, that the previous baking, varnishing, gilding, or soaking, of these woods in any melted matter, only tends to impair the property that renders them valuable. They should be simply rubbed on the outside with wax and a cloth. In pendulums of this construction the error is greatly diminished, but not taken away: but there are a considerable number of ingenious contrivances for entirely removing it.

The combination of metallic bars in the gridiron pendulum

pendulum seems to be the most simple and effectual contrivance for this purpose, and is therefore the only one we shall here describe. From the point of suspension *A* (fig. 39) proceeds a small flexible spring, by the alternate flexure of which the vibration is allowed to be made. The lower part of the spring is fixed to the frame *B C*, out of which proceed five equal cylindrical bars of metal. The two outer bars are steel, as is likewise the middle bar: the other two are a composition of zink and silver. The two outer bars are fastened in the piece *B C* by means of pins, but the three inner ones pass loosely into holes in the same piece, without being fastened at all. The three inner bars pass through a piece *D E*, near their upper ends, in which they are all fastened by pins. At the lower extremities of the bars is another cross piece *F G*, in which all the bars are fastened by pins, except the middle bar that passes freely through, and carries the ball or lens *H*.

The consideration on which this construction depends is, that a given increase of temperature will cause the bars of zink and silver to expand about twice as much in their linear dimensions as the bars of steel. To simplify our explanation, let us attend only to the ball *H*, without regarding the weight of the other parts. It will then only be required, that the distance between *A* and the ball *H* should continue unaltered in every change of temperature, which is accomplished thus. Imagine the

the outer bars of steel to expand by heat, and they will suffer the frame $F G$ to descend; the middle steel-bar will likewise expand in the same ratio of its length; and the ball H would consequently be removed farther from the point of suspension A if no other parts of the apparatus were affected by the heat: but the same heat expands the bars of zinc and silver much more in proportion to their lengths, and therefore the adjustment may be so made as that the expansion of these shorter bars may be equal to that of the longer ones of steel; that is to say, the expansion of these two bars may be such as to remove the cross piece $D E$ farther from $F G$, so as to raise the ball H upwards through a space exactly equal to the quantity of the expansion of the central, and the two outer bars. Whence the ball H will always be kept at the same distance from A .

It may be observed, that the two outer steel-bars answer the purpose of a single bar with regard to the expansion, as do likewise the two next adjacent to the middle bar. These bars, namely, two of steel and one of zinc and silver, would have been sufficient in theory, but the necessity of binding them together in that case produces a degree of friction of the parts that is, not without reason, thought to have a bad effect, by causing the action to take place by starts. The adjustment of the contrary expansions is made by pinning the piece $D E$ at the various distances from $F G$, by which means the ratio of the length of steel to that of the zinc and

and silver may be altered in any convenient degree: and this adjustment being made from actual observation of the clock's going, it is evident, that no practical inconvenience can result from our gratuitous supposition of all the parts, except H, being without weight.

C H A P. VIII.

OF THE MOTION OF A BODY, WHICH IS ACTED
UPON BY A CENTRIPETAL FORCE.

IF a body at A (fig. 40) be carried with an uniform direct motion in a given line A E, and rays be drawn from the equidistant points, A, B, C, D, E, to any point L, without the line A E, the areas A L B, B L C, C L D, D L E, &c. will be equal to each other *.

And these areas which are described in equal times, will not be altered by any centripetal force acting on the body A, and impelling it towards L. For, .

Suppose the body A to describe the equal spaces A B, B C, C D, and consequently with respect to the point L, the equal areas A L B, B L C, C L D, in equal times. Let a centripetal force be impressed at D, which singly would cause it to describe the space D d in the same time as D E, which is equal to D C, &c. Complete the parallelogram D d e E, and (23, s) at the end of the time the body will be found at e; having described the diagonal D e. The tri-

* This reasoning depends on that well-known proposition (Euclid I. 38) that triangles constituted upon equal bases, and between the same parallel, are equal to one another.

angle $d e l$ will then be equal to $d f l$, because both stand on the same base $d l$, and between the parallels $d l$ and $e c$. Continue $e f$ equal to $d e$, and the area $e f l$ will be equal to $d e l$, for the same reason; $e f$ representing the space which would be described in the same time as $d e$, if no new impulse were given at e . Let a force $e m$ be impressed at e , and by the same process it is proved, that $e f l$ is equal to $e f l$. The like may be proved of the triangles $f g l$, $g h l$, &c.

M Since therefore any single impulse can only alter the velocity and direction, but never affect the area described, it is plain that any number of successive impulses will likewise have no effect in altering the area. Suppose the number of impulses to be infinite, or, in other words, let a force directed to the center act continually on the body, and a polygon with an infinite number of sides, that is to say, a curve, will be described, whose radius accompanying the moving body, will describe or sweep over equal areas in equal times.

N And conversely, if a body revolve about a point, so as to describe a curve, whose radius accompanying the body, shall sweep over equal areas in equal times, the centripetal force which deflects the motion from a right line, must be directed to that point.

O But no instance of a centripetal force directed to an immoveable point is found in nature. Bodies attract one another, and that mutually. Therefore,

fore, if one body revolves about another, this last will not remain at rest, but will revolve in a * similar curve about the common center of gravity, as will also the first body. That is to say, if the center of gravity be at rest, the two bodies will absolutely move in similar curves about that center, and relatively about each other in curves similar to those last mentioned.

These motions will not be altered, if the center of gravity be supposed in motion (78, r. 79, w).

Therefore, when we speak of the orbits and periodical revolutions of bodies, we may in general regard one of the bodies as stationary, and the other as revolving round it.

If a body revolve round a center in an orbit which is not circular, it is plain, that to describe equal areas in equal times, it must move swifter when near the center than when more distant; and it is likewise evident, that when the velocity, and consequently the tendency to fly off in a tangent is increased, a greater centripetal force will be required to retain it in its orbit.

From the properties of the ellipsis it is demonstrated, that a body revolving in that curve, whose centripetal force tends to one of its foci, must in any part of its orbit be attracted towards that focus, by a force which is reciprocally as the square of its distance †.

* Principia, I. 57.

† Principia, I. 11.

- T A very complete and clear idea of the ellipsis may be had from the common way of describing it; if a thread $c A c$ (fig. 41) be fastened by its ends at the points $c c$, and a pointed instrument be inserted in the bight or bend at A , and moved towards B or E , keeping the thread at full stretch, it will in one revolution describe the ellipsis $A B D E$. $c c$ are called the foci.
- V To illustrate this doctrine of revolving bodies, we may observe, that as gravity constantly acts on all bodies in the vicinity of the earth, attracting them towards its center, every projectile, which is not thrown in the line of the perpendicular, may be considered as a body revolving about that center; and if its orbit be not sufficiently large to contain or circumscribe the body of the earth, it will be interrupted in its course, and remain at rest somewhere on the surface. Thus let $A B E$ (fig. 42) represent the earth, whose center is at c ; then if a body be projected from A in the direction $A F$, it will by the action of the centripetal force, be deflected into the curve $A G B$, and will remain at rest at B , being prevented from describing the whole orbit $A G B D A$, by the body of the earth, which interrupts its course at B . But the part $A G B$ of the elliptical orbit of a projectile is so small, in comparison to that part which is not described, that it may without any sensible error be considered as a parabola, except so far as the resistance of the air, which is not here regarded, makes it fall short of B , by destroying part of its motion.

The orbit $A G B D A$, of which the parabola is v part, would have been described upon the supposition, that the attraction towards the center continues to observe the same law within as without the sphere. But this supposition, however, is not true; for a sphere of uniform density, composed of particles which attract each other with forces reciprocally as the squares of their distances, will attract bodies without its surface according to the same law; relation being had to its center. But the centripetal forces of bodies placed within the sphere, will be directly as their distances from the center*.

Let the circle $B C D G$ (fig. 43) represent the earth. From the top of the mountain A , let a body be projected in the horizontal direction $A r$, with a force that will carry it to B on the surface. Imagine it to be projected in the same direction with a still greater force, and it will be carried to C . A still greater increase of force will carry it to D . And a yet greater augmentation will carry it round the earth to A , where it will proceed with a velocity equal to that with which it was first projected, and by consequence, the resistance of the air being disregarded, will revolve in that orbit for ever. But if the projectile force be still more increased, it will describe the ellipsis $A b A$ with an unequable motion; slower at b and swifter at A , and continue to revolve for ever in that orbit.

*-Principia, I. 82.

If gravity acts in the distant spaces of the heavens inversely according to the squares of the distances, it will be easy to apply this to the motions of the celestial bodies. We shall again resume this subject; but in the mean time it is necessary, that the appearances should first be described before an explanation of them can be given.

B O O K I.

§ E C T. III.

Astronomy.

C H A P. I.

CONCERNING THE SYSTEM OF THE UNIVERSE.

THERE are two methods by which knowledge may be acquired and announced; namely, by analysis, or by synthesis. In the method of analysis the process is made from things that are compounded to things that are more simple. Causes, or first principles, are investigated by attending to, and examining their effects. But in the method of synthesis the process is directly the contrary; for here the causes, or first principles, being known or assumed, are made by composition or combination to account for their effects. It is very manifest, that in the acquisition of philosophical knowledge the former method must be first made use of. We see no simple events in nature, and we cannot come at causes but by analysing the effects we behold. Thus it is that first prin-

ciples are obtained, which may afterwards be extended by synthesis, to account for other phenomena in a more universal manner. Generally speaking, the analysis appears best adapted for acquiring knowledge, while the synthetical method is more convenient and concise for communicating it, when known.

In the communication of the knowledge that relates to the heavenly bodies, and is termed astronomy, we might assume as established first principles every thing which respects their mutual positions and motions absolutely considered, and from thence deduce synthetically the phenomena that would appear to a spectator placed on the earth or elsewhere. Or we might still more generally, from the laws of motion assumed as first principles, deduce the consequences that would arise from the motions of bodies in circumstances such as the heavenly bodies are known to be placed in. By these methods the science would be most expeditiously taught, and even supposing the first principles to be merely assumed at hazard, or gratuitously, yet their constant agreement with the events that happen in nature might furnish no inconsiderable presumption of their truth. However, it is clear, that the knowledge on which the synthetical reasoning is built, must be either assumed gratuitously, or obtained by analysis. It is not easy, nay, it is perhaps impossible, to form any conception how a finite intelligence can, to any advantage, make

make use of first principles taken without a previous use of the analytical method of deduction. Yet the whole history of natural philosophy affords numberless instances of men of real abilities who have indulged their vanity and indolence in deducing consequences from principles entirely hypothetical, and often false. Far from wishing to imitate these, we shall not assume, even established truths, without giving their proofs where they can be explained with that degree of facility which our intention demands. Instead, therefore, of deducing the apparent phenomena from the real motions of the heavenly bodies, it is presumed that it will be much more interesting, though rather more prolix, to note the obvious appearances, and thence infer their causes. These inferences have been the consequence of the observation and study of several of the most distinguished men of genius in the course of many ages. The truth has been acquired by slow degrees, and indirect methods.. It has often been obscured by the admixture of error. Its progress has been retarded by the operation of prejudice, and the pride of false science. When discoveries are completed, it is easy to trace the most direct steps by which they might have been made, though it has scarcely ever happened that the discoverers proceeded by those steps. Let us therefore pass over in silence the various and intricate schemes made use of to solve the celestial appearances before the ancient system of the world

was revived by Copernicus*, and since established for ever by the immortal Newton. Let us imagine ourselves in the open air busied in the contemplation of the phenomena that occur in the heavens, and while we note the facts, and make plain deductions from them, we shall be insensibly led to the knowledge of the beautiful regularity and order which prevail through the immense regions of space, and evince the intelligence and power of the First Cause.

The first and most obvious phenomenon that presents itself to observation, is the apparent diurnal motion of the visible sphere of the heavens; by which the sun, moon, and stars are seen to rise and set. This motion is observed to be subject to seeming irregularities. If its period be estimated from sun-rise to sun-rise, a little time shews, that the sun does not always rise at the same point, nor remain above the horizon so long in winter as in summer. The moon is still less adapted to the purpose of determining this period, its variations being in every respect more conspicuous. The stars remain, which appear indeed to rise, and set regularly, but yet in a period shorter than the natural day; for those

* A. D. 1543, the year of his death, After suppressing his book, "*de Revolutionibus orbium celestium*," for more than thirty-six years, it was at length published, and a copy brought him a few hours before his death. Gassendus in *vita Copernici*. See also Sir John Pringle's elegant "*Dissertation on the Attraction of Mountains*."

stars, which at a certain time of the year are seen to rise at midnight, are found to make their appearance early in the evening, after the space of three months is elapsed. It is therefore to be determined which of those motions ought to be regarded as the motion of the heavens; and it is much more obvious and intelligible, to suppose, that the sun, by a relative motion to the eastward with respect to the fixed stars, should make the days somewhat longer than the real time of a revolution, than that all the stars, while they preserve their mutual distances unaltered, should constantly move with a velocity greater than that of the supposed celestial sphere. To determine this relative path of the sun is not difficult. By the shadow of a perpendicular staff or other equivalent instrument at mid-day, its varying declination towards the north or south may be known, and the advance in the rising of the stars will mark its difference in right ascension. By this, or some such method, it may be discovered, that while the fixed stars rise and set, each on its proper points of bearing or position, without varying their relative situations; the sun, by describing annually a circle towards the east, inclined to the direction of its daily course in an angle of $23\frac{1}{2}^{\circ}$ * degrees, must occasion all the difference of seasons, length of days, &c.

* Every circle is supposed to be divided into 360 parts, which are called degrees.

- B** In noting these appearances, it is natural to select the brightest stars as objects of our attention. The planetary bodies will on this account be the early objects of our notice. The planet Venus especially, receding from the sun to the eastward, will appear as an evening star in the west after sunset; and afterwards will disappear on its re-approach to it, and be seen at a nearly equal distance to the westward, and rising before the sun, become a morning star. The slowness of its apparent motion near its greatest elongation or angular distance from the sun, shews that it is moved in an orbit, near the center of which the sun is placed; and the short time employed in passing from its greatest elongation eastward to its greatest elongation westward, when compared with the time of its course between the same elongations in the contrary direction, shews that its revolution is made from west to east. The proportion between its distance from the sun, and that of the sun from the earth, may be found from the quantity of its greatest elongation.
- C** To illustrate this, let s (fig. 44) represent the sun, z the earth, $imkn$ the orbit of Venus, csd part of the ecliptic, or sun's apparent annual path in the heavens. Then to a spectator at z , situated nearly in the plane of the planet's orbit, the planet when at B will be referred to the point b in the ecliptic; and when by its absolute motion in its orbit, it has described the arc Bv , it will appear to have described the arc bD . When at v , it will appear stationary at D , and after a little time begin

to move back from d to b : for after describing the arc $u j$, it will again be seen at b . Continuing its course, it will arrive at v , having apparently passed through the arc $b c$. At v it will again become stationary and afterwards move to g , f , a , &c. which will be represented by their corresponding points in the ecliptic. Now, since the motion, as seen from the earth, is that which appears in the ecliptic, and since the apparent motion from b to p may as well be produced by a real motion from j to u , as by one from b to u , it remains to be determined in what direction the real motion is made by which the apparent motion is produced. Now, because $p c$ and $e d$ are tangents to the orbit, the points d and c , which correspond with the positions u and v , are those of its greatest elongations; and because the arc $u v$, which is passed over in the inferior part of the orbit between the two greatest elongations, is less than the superior arc $v a b u$, which is passed over between the same elongations, it is plain, that when the planet is in the inferior part of the orbit, the space $c d$ will be performed in less time than when it is in the superior part. It is also evident, when the planet moves in the superior part of its orbit, that the apparent motion in the ecliptic has the same direction as the real motion. Therefore, since we have a criterion to distinguish the motion in the superior from that in the inferior part, we can easily determine the direction of the motion in its orbit, which is proved to be from west to east,

- 2 The distance of Venus from the Sun, in proportion of that of the Sun from the earth is determined from its greatest elongation: thus draw the line vs , which will be at right angles to the tangent ve ; then in the right angled triangle ves , by the rules of plain trigonometry,

As radius

Is to the sine of the angle of greatest elongation ves ,

So is the Sun's distance from the earth es ,

To the distance of Venus from the Sun vs .

- 2 By similar observations on the planet Mercury, it is determined that its revolutions are performed round the sun in the same manner, because they are accompanied with circumstances of the same nature as appear in the motion of Venus.

C H A P. II.

OF THE FIGURE AND MOTION OF THE EARTH.

WE have not yet considered the effects which the sun's annual motion in the ecliptic has upon the apparent motions of the planets, though it is very considerable: neither have we determined whether this apparent annual motion be the consequence of a real motion of the sun about the earth, or of the earth round the sun. The celestial phenomena may be explained either way, but in a much more simple and intelligible manner by the latter supposition. We shall therefore previously give an account of the figure of the earth, and the reasons on which the supposition of its motion is founded. The proof will come more properly when we treat of the physical causes of these motions. At present we only describe appearances, and draw plain inferences from them.

The purposes of astronomy require, that the fixed stars should be classed into constellations. When their relative situations are known, it must soon be perceived, that their diurnal revolutions are performed round an axis, obliquely situated with respect to the horizon, or circle that bounds our view; one of its extremities or poles being above the horizon to the * north, and the other as far below it to the

* In this elucidation the observer is supposed to be in north latitude.

south, and consequently invisible. By travelling to the northward, the north pole is observed to become more elevated, and that exactly in proportion to the space travelled over; from which circumstance it follows, that the earth is round or spherical.

H For, let IBE , fig. 46, represent a plain section of the earth at right angles to its surface; let AB represent the plumb line or perpendicular, and BP the line of direction, in which the * pole star is seen; the angle ABP will then be the complement of the altitude of the star: let the same star be seen from another point E in the line of direction EQ , which, by reason of the great distance of the star, may be esteemed parallel to BP ; the angle DEF will then be equal to the angle ABP , or complement of the former altitude at B . From E draw the plumb line or perpendicular ER ; the angle DER will then be the difference between the two co-altitudes ABP , FEQ , or between the two altitudes; but this difference is equal to the angle BCR , formed between the plumb lines, and is proportional to the arc or distance BET .

Now,

* There is no star situated at the pole. The star α in Ursa Minor, which is called the pole star, is about $2\frac{1}{2}$ degrees distant from it.

† Let the part HN of the line OP (fig. 46) be comprehended between the perpendiculars AN , PN , which, if continued, meet at G . Conceive HN to be divided into an indefinite number of equal parts, by means of perpendiculars BI , CK , &c. severally prolonged till they meet. Then in the triangle HGI , the sides, HG , GI , are equal, because, opposed to equal angles at H and I : and for a like reason, in the triangle

Now, if the angle formed between two perpendiculars to a given line be always in proportion to that part of the line comprehended between the perpendiculars, the line itself is circular: and that solid, whose sections, passing through two opposite points, are all circles, is itself a sphere.

Hence, if the length of the arc BE be measured, and its quantity in degrees known by observation on a star, the length of the whole circumference of the earth may be found by this proportion. As the quantity of degrees is to the length measured, so is the whole circumference, or 360 degrees to its length.

The modern circumnavigation likewise proves the sphericity of the earth; for, by sailing continually eastward, or continually westward, vessels arrive again at the port from whence their first departure was taken.

Also, in an eclipse of the moon, the shadow of the earth is always projected in a circular form. Now, it is evident, that the body, whose shadow is in all positions a circle, must itself be a globe.

Unfurnished with those proofs, which the sagacity and more accurate observations of later

angle ICK , the sides IG , CK are equal. The same process of argumentation will extend to prove, that all the small triangles between u and n are isosceles, having a common vertex at G ; that is to say, there is no definite part of un from which a right line can be drawn to the point G , either greater or less than uG : un is therefore a curve, having such a relation to a certain point, that all right lines drawn from it to that point are equal; or it is circular. Which was to be shewn.

ages have afforded, the ancients could not adduce those reasons for the earth's motion, that depend on the general laws of motion, and the nature of gravity. Without doubt they had recourse to those which depend on the moral fitness of things. They were persuaded that the wisdom of the Creator had formed every thing in the best manner possible, and therefore, that when an effect could be as well produced by simple as by complicated causes, the observer of nature ought to attribute it to the former. They saw the two planets, Mercury and Venus, revolving round the sun in orbits, whose radii are less than the distance between the sun and the earth: the superior planets, Mars, Jupiter, and Saturn, were also observed to move in orbits about the sun, but at greater distances than that between the sun and earth. If the sun were supposed to move absolutely in the ecliptic or its apparent path, it must carry the orbits of these bodies along with it, and consequently their absolute motions must be very complicated; but if the earth be supposed to describe an orbit round the sun, between Venus and Mars, the absolute motions become simple and natural, and an admirable uniformity prevails throughout the system.

- N The annual motion of the earth being allowed on this principle, its diurnal motion would follow by the same argument; it being much more reasonable and consistent to suppose, that the earth, by a daily revolution on its own axis from west to east, should occasion the apparent motion of the celestial bodies, than

than that those bodies should, besides their other various motions, have that astonishing velocity which a real diurnal motion would produce. The objections common observers might make would be easily disproved by men whose penetration was capable of going thus far. From the observations by which the spherical form of the earth was discovered, they would also gather, that bodies fell not absolutely down, or in a direction referable to pure space, as was imagined, but always in a line directed towards the center of the earth, and consequently that no danger of bodies falling off would arise from its continual change of position. The instances of ships carried by the tides in calm weather would likewise serve to shew, that the relative motions or positions of bodies are not changed by an equal velocity given to them in the same parallel direction.

C H A P. III.

OF THE MUTUAL APPEARANCES OF THE SUPERIOR AND INFERIOR PLANETS.

THAT the planets Mars, Jupiter, and Saturn re- o
volve in orbits, which include the orbit of the
earth, is evident, because they are frequently seen
in the part of the ecliptic directly opposite to the
sun; and that the orbits respect the sun as a center,
appears as well from those oppositions which happen
in every part of the ecliptic, as from their unequal

apparent motions, which are explained by referring them to that center.

P We have considered the apparent motions of the inferior planets as far as relates to their situation with respect to the sun. The motion of the earth affects those appearances, to speak in general, only by prolonging the time they employ to return again to the same situation.

Q The earth at z (fig. 44) is a superior planet with respect to Venus. A spectator on Venus at b would see the earth z elongated from the sun under the angle zbs ; which angle of elongation would increase by the motion of Venus in its orbit from b to v , where it becomes a right angle zvs . From j it would be seen in an angle of still greater elongation zjs , and from m it would be seen directly in opposition to the sun. Passing from m to κ , v , &c. the angle of elongation would decrease till the arrival of Venus at n , whence the earth would be in conjunction with the sun, and the angle of elongation would vanish. This relative motion of the superior planet with respect to the sun is contrary to the order of the signs, or from east to west, and depends entirely upon the motion of the inferior planet on which the spectator is supposed to be placed.

R If the earth z was at a distance indefinitely great, the lines $z s$, $u s$, $j s$, &c. might be esteemed parallel, and consequently the spectator would behold it always in the same point of the ecliptic, its situation with regard to the sun being varied

only

only by the apparent motion of the sun, occasioned by the real motion of Venus. But as this is by no means the case, an apparent motion of the earth among the signs of the ecliptic will be produced. Thus, the earth viewed from *N*, will appear among the fixed stars at *P*; from *B* it will appear at *R*; from *U* at *O*, where it will be stationary so long as the orbit of Venus does not sensibly differ from its tangent; from *J* it will be seen returned back to *R* with a retrograde motion; from *M* at *P*; from *K* at *T*; from *V* at *Q*, where it again becomes stationary; and from *A* it will be again seen at *T*, its motion having again become direct: whence we may observe, that

When a superior planet viewed from an inferior *s* appears stationary, the inferior planet viewed at the same time from the superior is also stationary; and,

When the inferior planet viewed from the superior *t* moves apparently retrograde, or contrary to the order of the signs, the superior planet has also an apparently retrograde motion.

But since the earth has an annual motion round *u* the sun in its orbit, (*110, M*) we are therefore to discover what part of the apparent motion of Venus is produced by that cause. It is plain, that if the earth were at rest, and Venus seen at *u*, its greatest elongation, it would again be seen in the same position, after performing a complete revolution in its orbit. But while Venus is performing this revolution, the earth is carried from *s* towards *w*, and so

forth. Therefore Venus must pass between two similar elongations, not only a complete revolution, but likewise the whole angular space which the earth has performed in the same time. Hence its periodical time may be found. For the time between two similar positions is observed to be 583 days. Now, dividing the earth's orbit into 365 equal parts or days, the angular velocity of Venus will be denoted by the angular space passed over in the given time, namely, one revolution, or 365 days added to 583 days, equal to 948; and the earth's angular velocity will be 583.

- v The periodical times of Venus and the earth will be reciprocally as their angular velocities; consequently,

As the angular velocity of Venus - 948
 Is to the angular velocity of the earth 583
 So is the periodical time of the earth 365
 To the periodical time of Venus 224½

- w Were it not for the fixed stars, it would be impossible to discover or observe the annual motion of the earth. We should conclude, that each planet made a complete revolution between any two similar situations with respect to the sun, because the spaces of elongation are similarly described, and are in quantity the same, whether the earth be in motion or not. Thus, if the earth be fixed at z, the same apparent elongations will be made by Venus with any velocity whatsoever in its orbit, but they will occur more frequently the greater the velocity. If a motion be given to the
- 7
- earth

earth in the orbit EW , Venus will approach from U to M , which is now in motion, with a velocity equal to the difference between its angular velocity and that of the earth: or if the earth's angular velocity be greatest, it will apparently recede from M , and describe its revolutions in the contrary direction to its real motion. Now, as all the apparent motion of Venus in elongation is known by its approach or recess from the line SE , and since any angular motion of SE can only change the relative velocity of Venus; and since a change of velocity will not alter the elongations, except as to time, it is evident, that we cannot determine whether E be at rest or no, from the appearances of the planets which revolve about the sun. It is then from the apparent motion of the sun, with respect to the fixed stars, that we conclude that the earth describes an orbit in about 365 days.

If the superior planet E be at rest, the retrograde x motion of the inferior planet U among the fixed stars will be the same as its motion in elongation, viz. the angle $U S V$. But if E move in the same direction as U , but angularly slower, the arc described by the retrograde motion in the ecliptic will be less than that described between the two opposite elongations. The same is true of the retrograde motion of the superior viewed from the inferior planet.

For the motion of E towards W causes an apparent motion of the sun towards B . And as the retrograde motion of U referred to the arc DS is

slowest near the elongations, it is plain that v will not become stationary in the ecliptic till its apparent motion in elongation from D towards s is equal to the sun's apparent motion in the contrary direction; that is to say, till some time after passing the greatest elongation, suppose at d . After which its motion must become retrograde till it arrives at H , equidistant from its greatest elongation on the other side, where it will again become stationary, its apparent motion in elongation being equal and contrary to that of the sun in the ecliptic. Now, the angle HED is less than the angle of retrograde motion in elongation CED . And since the angle IEQ is equal to HED , it is also less than CED . But those angles IEQ and HED are the measures of the retrograde motions of the superior and inferior planets, when viewed from each other. Wherefore the proposition is evident.

* C H A P. IV.

OF THE SUPERIOR PLANETS, AND OF THE TRUE FORM OF THE PLANETARY ORBITS.

- 2 THE appearance of the earth when viewed from Venus being explained, it will be easy to apply that explanation to the apparent motions of the superior planets. Of the two inferior planets Venus served us as an instance; and of the three superior ones we shall select Jupiter, as being the most bright and conspicuous. The motions of this planet being accounted for, similar observations and similar reasoning will obviously solve those of the other planets,

nets, whose particular phenomena will not, therefore, require a more minute elucidation.

That the planet Jupiter revolves in an orbit, which includes that of the earth, and respects the sun as its center, was shewn in the beginning of the last chapter; and its apparent motions are observed to be similar to those which it was proved the earth would have when seen from Venus. It remains to discover its periodical time and distance from the sun.

Let s (fig. 47) represent the sun, e the earth, j Jupiter, the circle eca the earth's orbit, and the circle jjA the orbit of Jupiter. Suppose Jupiter to be in opposition to the sun. The earth revolving in its orbit will, in the space of 365 days, arrive again at e , but the opposition will not then happen, because Jupiter in the mean time will have moved in its orbit towards j . The earth must therefore pass through the arc ec or $33\frac{1}{2}$ days before it overtakes it. Consequently, the angular velocity of Jupiter will be denoted by $33\frac{1}{2}$, and that of the earth by one whole revolution, (or 365) added to $33\frac{1}{2}$, equal to $398\frac{1}{2}$. But as the periodical times are reciprocally as the angular velocities, it will be

- * As the angular velocity of Jupiter $33\frac{1}{2}$
 Is to the angular velocity of the earth $398\frac{1}{2}$
 So is the periodical time of the earth 365 days
 To the periodical time of Jupiter 4340 days.

* Smaller fractions being rejected, the periodical times are not here exact.

c The periodical time of Jupiter being thus obtained, it will be easy to determine its * heliocentric place at any time before or after the opposition, and the proportion of its distance from the sun to that of the earth from the sun being known, its † geocentric place may likewise, at any time, be discovered. Its proportional distance is thus found.

d The figure as before. Suppose the earth to have moved from x to c , in a given time. From the time may be found the quantity of the angle $xs c$; and in the same time Jupiter will have moved to b , the angle $js b$ being also known from its proportion to his whole periodical revolution. Subtract the angle $js b$ from the angle $js c$, and the remainder will be the angle $bs c$. By observation find the angle bcs , or Jupiter's elongation from the sun. In the triangle csb , the sum of the two angles bcs and bsc being taken from 180 degrees, leaves the angle csb . Then, by plain trigonometry,

x	As the sine of the angle of the earth's elongation, when viewed from Jupiter	csb
	Is to the sine of the angle of Jupiter's elongation, when viewed from the earth	sbs ,
	So is the earth's distance from the sun	cs
	To Jupiter's distance from the sun	bs .

f The angle of the earth's elongation, when viewed from Jupiter, is called Jupiter's annual parallax,

* Viewed from the sun as a center.

† Viewed from the earth as a center.

and

and is always equal to the difference between its heliocentric and geocentric place in the ecliptic, as a little consideration will shew.

By similar observations on the other superior planets, it is found that their apparent motions are attended with circumstances of the same nature as those of Jupiter. The same consequences must therefore follow respecting their orbits, periods, and other affections.

Thus far we have spoken of the appearances of the planets, as if their revolutions were performed in circular orbits, in the center of which the sun was supposed to be placed. But this is not the case. Conjunctions, oppositions, similar elongations, or other mutual situations of the planets, do not return again in exactly the same time, and their distances from the sun are found to be greater or less in different parts of their orbits, their angular velocities being always greater when the distances are less. Thus, by the increased diameter of the sun during the winter half-year, we find that the earth's distance is diminished; and that its velocity is increased, is evinced from the apparent motion of the sun, by which it passes through the winter half-circle of the ecliptic in near eight days less than it employs to describe the summer-half. By a variety of observations of elongation or parallax, the relative or proportional distances of the planets from the sun, and their velocities are found for every heliocentric position. Whence they are proved to revolve in elliptical orbits, the
sun

sun being placed in one of the foci; and their velocities are such, that a radius drawn from the sun to the planet, and supposed to move with it, describes equal areas in equal times.

L - The distance between the center s (fig. 41) and one of the foci c of an elliptical orbit, is called its eccentricity. The two extreme points of the transverse or longest diameter, L and U , are called the apsides. If the focus about which the equal areas are described be at c , the point L nearest that focus is called the lower apsis, and U is called the upper apsis; the diameter UL being called the line of the apsides. But it is more common to say, that a planet is in its perihelium when at L , and in its aphelium when at U . When the earth is in its perihelium, the sun is said to be in its perigee, and when the earth is in its aphelium, the sun is said to be in its apogee.

M From the generation of the ellipsis (97, T) it is evident, that the whole length of the string is equal to the transverse diameter: because, when the point A is at U , the return of the string from U to the nearest focus c , is exactly equal to the part between L and the other focus c , where there is no string. Let GH be the conjugate, or shortest diameter of the ellipsis, and CG will be equal to half the string, or half the transverse diameter. Suppose now a planet to revolve in the ellipsis about the focus c nearest L . Then CL will be its nearest distance, CU its greatest distance, and CG its mean distance. For, CG equal to SL , exceeds

ceeds the least distance cl by the quantity cs , which is just as much as it falls short of cu , the greatest distance. If therefore the mean distance cg , and the eccentricity cs of a planet be given, its orbit may be described: because the greatest distance is equal to the sum of the eccentricity and the mean distance; and the least distance is equal to their difference.

The eccentricities of the planets are so small, n that their orbits approach nearly to circles.

If the plane of the earth's orbit were extended o indefinitely every way, it would mark that circle in the heavens which is called the ecliptic, or sun's path. If the orbit of any other planet be situated in this plane, it will always be seen in the ecliptic, whether viewed from the earth or the sun. But if the plane of the planet's orbit be obliquely situated with respect to that of the ecliptic, it will intersect it in a line passing through the center of the sun, and the planet will never be seen in the ecliptic but when in the points of intersection. These opposite points of the ecliptic are called the nodes, and the line of intersection is called the line of the nodes. When a planet crosses the ecliptic from south to north, the node is termed the ascending node; and when it crosses from north to the southward, the node is termed the descending node.

The orbits of all the planets are inclined to the p ecliptic in small angles.

C H A P. V.

OF THE AFFECTIONS OF THE PLANETS.

Q By supposing ourselves in the place of one of the ancients who discovered the order of the planetary system, we have displayed in a cursory manner some of the most obvious phenomena, and pointed out their natural consequences. What has been said is sufficient to shew to those who are totally unacquainted with the subject, that the doctrine of a system of bodies revolving round the sun is not merely ideal, but founded on the most natural deduction from the celestial appearances. For the processes by which the planets places are determined in elliptical orbits, we refer the reader to treatises written expressly on the subject, and in the mean time proceed to note several of those affections of the heavenly bodies, as determined by the accurate observations of modern times.

R Seven planets, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgium Sidus*, revolve about the sun in orbits included within each other, in the order here used in mentioning their names, Mercury being nearest the

* This planet was discovered in the year 1781, by William Herschel, Esq. a native of Hanover.

Sun. These are called primary planets, besides which, there are fourteen which are called secondary planets, Moons or Satellites. The secondary planets respect the primary planets, performing their revolutions about them, but are at the same time carried round the Sun in the orbit of the primary. Saturn is attended by seven Moons, Jupiter by four, the Georgium Sidus by two, and the Earth by one, all which, except the last, are invisible to us, by reason of their smallness and distance, unless telescopes be made use of. Without this instrument, it would likewise be impossible to ascertain the apparent diameters of any of the celestial bodies, the Sun and Moon excepted. The following table exhibits some of the affections of the primary planets.

Anno 1784.	MERCURY.	VENUS.	EARTH.
Greatest possible elongation of inferior and parallax of superior planets, - }	28°. 20'	47°. 48'	* *
Proportional mean distances from the sun, - }	38710	72333	100000
Periodical revolutions,	87d. 23 h. 15½ m.	224 d. 16 h. 49½ m.	365 d. 6 h. 9½ m.
Diurnal rotations, - -	Unknown.	23 h. 22. m.	23 h. 56 m. 04 f.
Inclinations of their orbits to the ecliptic, - }	7° 30'	3° 23 ½'	* * *
Eccentricities, - -	7960	510	6180
Place of the ascending node, - - }	1 f. 15 deg. 46½ m.	2 f. 14 d. 34 m.	* * *
Place of the aphelium,	8 f. 14 d. 13 m.	10 f. 2 d. 38 m.	9 f. 9 d. 15½ m.
Proportion of axis to the equatorial diameter, - }	Unknown.	Unknown.	2289 to 2300
Greatest apparent diameters, - - }	11"	58"	*
Diameters, if seen at the sun's mean distance, or proportional diameters, }	7"	18". 7	17". 3
Diameters in geographical miles; that of the sun being 762490, - }	2782	6637	6875½
Mean distances from the sun in semidiameters of the earth, }	9210	17210	23799
Mean distances from the sun in geographical miles, - }	31671900	59181700	81818400 <i>81818400</i>
Proportions of light, -	668	191	100

MARS.	JUPITER.	SATURN.	GEORGIUM SIDUS, 1782.
47°. 24'	11°. 51'	6°. 29'	3°. 44'
152369	520098	953937	1903421
656 d. 23 h. 30 $\frac{1}{2}$ m.	4332 d. 8 h. 51 $\frac{1}{2}$ m.	10761 d. 14 h. 36 $\frac{1}{2}$ m.	30445 d. 18 h.
24 h. 39 m. 22 s.	9 h. 56 m.	10 h. 16 m. 0,4 s.	Unknown.
1°. 51'.	1° 19 $\frac{1}{2}$ '	2°. 30 $\frac{1}{2}$ '	46' 12''
14218	25277	53163	4759
1 f. 17 d. 59 m.	3 f. 8 d. 50 m.	3 f. 21 d. 48 $\frac{1}{2}$ m.	3 f. 13 d. 1 m.
5 f. 2 d. 6 $\frac{1}{2}$ m.	6 f. 10 d. 57 $\frac{1}{2}$ m.	9 f. 0 d. 45 $\frac{1}{2}$ m.	11 f. 23 d. 23 m.
1272 to 1355	13 to 14	10 to 11.	Unknown.
25''	46''	20''	4''
9'' 1	193'' 7	171'' 7	74'' 5
3617	76982	68238	34217
36262	113778	227028	453000
124666000	425536000	780395100	1557350000
43	3 7	1. 1	0. 276

C H A P. VI.

OF PARALLAXES, AND OF THE TRANSIT OF VENUS.

- S** ONE of the most usual methods of measuring inaccessible distances, is by means of two stations whose distance from each other is known, and the angles formed at each station between lines supposed to be drawn from the distant object to them, and the line that joins the stations to each other. Thus the distance between A and B (fig. 48) being known, as likewise the angles CAB and CBA , the distance AC or BC may be readily found by plane trigonometry.
- T** Suppose the object c , when viewed from B , (fig. 48) to coincide with another object s , which is at a distance indefinitely great; then the object c will not appear to coincide with s , when viewed from A . For s , on account of its great distance, will be seen in the line As , parallel to Bs , and c will be seen in the line Ac , the angle sAc being the difference between the apparent places from A and B . This angle, because of the parallels As and Bs , will be always equal to the angle ACB , and is by astronomers called the parallax. It is usually distinguished by some appellation relative to the nature of the line AB : for instance, it is called the annual parallax, when AB is the radius of the annual orbit; the diurnal parallax, when AB is the semidiameter of the earth, &c.

It

It is scarcely necessary to observe, that the longer u the line AB is in proportion to the distance of c , the greater the angle ACB , and that in most practical cases, the greater the angle the less is the distance affected by any small error. It is therefore requisite that the base AB be as large as possible or convenient.

The distances of the planets were found by trigonometry, the distance of the earth from the sun being assumed as a base. But as that base cannot actually be measured, the distances are only proportional or relative, the base being supposed to be divided into 100000 equal parts; but whether those parts be miles, leagues, or answer to any other denomination of length, was not determined. The real distances must be discovered by a parallax whose base is known.

The diameter of the earth is in general used as w the base for determining the distances of celestial objects by their parallax, which parallax is found as follows.

Let AOB (fig. 49) represent the earth, c its center, and z the zenith or point in the heavens, situated perpendicularly over the point o at its surface. Then CH will be the rational horizon, and OK the sensible horizon. Suppose a spectator at c views a celestial object at z , the revolution of the earth will cause it to move apparently through the quadrant zH in six hours, at the end of which time he will see it in the horizon at H . But to a spectator at o it will ap-

pear in the horizon when at κ , after passing through the apparent quadrant or right angle $z o \kappa$, in a time as much less than six hours as the arc $z \kappa$ is less than $z h$, or 90 degrees. Hence the time of an object's passing between the zenith and sensible horizon being known, the angle $o \kappa c$, or horizontal parallax may be found. For, as six hours is to 90 degrees, so is the time observed, to the arc $z \kappa$, which being taken from 90 degrees, leaves the arc κh measuring the angle $\kappa c h$, which is equal to $o \kappa c$, or the horizontal parallax.

- y The horizontal parallax being discovered, the distance of the object follows by this analogy; in the triangle $o \kappa c$.

As the horizontal parallax,	fine	-	-	$o \kappa c$
Is to the earth's semidiameter		-	-	$o c$
So is radius	-	-	fine	90°
To the distance	-	.	-	$c \kappa$

The fixed stars have no parallax, either horizontal or even annual, whence it follows, that their distances are beyond all comparison greater than that of the earth from the sun.

- z It is obviously unnecessary in observations of parallax to wait till the object has described the whole apparent quadrant $z \kappa$ (fig. 49): for, when it is arrived at ϵ , the angle $z c \epsilon$ may be known from the time, and $z o \epsilon$ from observation, and their difference will be the angle of parallax $o \epsilon c$: so that in the triangle $o \epsilon c$ are given two angles, and the side $o c$, from whence the other parts are easily found.

Observations of parallax are conveniently made \mathbf{z} by the help of the fixed stars. Thus, if the object \mathbf{z} when at \mathbf{z} be seen in a given position with respect to a fixed star, it will continue to have the same position when arrived at \mathbf{z} , provided the spectator be at \mathbf{c} ; but if the spectator be at \mathbf{o} it will be seen depressed below its former position by the quantity of the parallax $\mathbf{D O z}$, because the star has no parallax, but is seen in the same apparent place either from \mathbf{o} or \mathbf{c} .

The Sun's parallax is so exceedingly small, that \mathbf{c} the best instruments in the hands of the most skilful observers, have scarcely effected more than to shew that it has one. To remedy this, the horizontal \mathbf{D} parallaxes of the nearer planets have been attempted, particularly of Mars, when in opposition to the Sun, this planet being then as near again to the Earth as the Sun is, and has therefore a parallax twice as great. But as this parallax is not found to exceed half a minute of a degree, the unavoidable uncertainty of observation, and other causes, render it not sufficiently exact to determine the Sun's distance within a 30th part of the whole. It is easy to comprehend how the Sun's distance may be found when the distance of Mars, from the Earth, in opposition, is known. Thus, if \mathbf{s} (fig. 50) be the Sun, \mathbf{z} the Earth, and \mathbf{m} Mars, in opposition, then \mathbf{zm} will be the distance of Mars from the Earth, and also the difference between \mathbf{ms} and \mathbf{zs} , or the respective distances of Mars and the Earth from the

Sun. The proportional distances are known. Therefore,

As the difference between the proportional distances of Mars and the Earth from the Sun,

Is to the proportional distance of the Earth from the Sun;

So is the distance between the Earth and Mars in opposition, or the difference between their real distances from the Sun,

To the Earth's real distance from the Sun.

Several other methods were devised by the ancients for discovering the Sun's parallax, which, though they shew the sagacity and penetration of their inventors, are less sufficient for the purpose than the foregoing. We shall therefore omit mentioning them, and give a short explanation of that for which we are indebted to the great Dr. Halley, by which the distance of the Sun is determined with greater accuracy than by any other method.

The planet Venus, as has been shewn, passes the Sun twice in revolving from any position of elongation to the same position again (105, c). At those times this planet is said to be in conjunction with the Sun.

When the planet Venus is situated in a line between the Sun and the Earth, it is said to be in its inferior conjunction; and when it is in the opposite part of its orbit, the Sun being in a line between it and the Earth, it is said to be in its superior conjunction. If the orbits of the Earth and Venus

were

were in the same plane, it is evident that Venus would pass behind the Sun with a direct motion every superior conjunction, and would pass over its *disc, or before it, with a retrograde motion every inferior conjunction. But as Venus's orbit is inclined to the ecliptic in an angle of about $3\frac{1}{2}$ degrees, this planet will, in general, pass to the northward or southward of the sun, and will only be visible on its disc when the inferior conjunction happens at or near one of the nodes. This happens but once (or sometimes twice at an interval of about 8 years) in more than 120 years,

To shew how this transit is applied to the purpose of finding the Sun's distance, we shall pass over those elements that enter into the computation previous or subsequent to actual observation, and shall only explain the general principles on which the method is founded.

Let s (fig. 51) represent the Sun, e the Earth, v, u, w , the planet Venus in different positions, the arc LN a part of the Earth's orbit, and the arc OM a part of the orbit of Venus. Then, because the angular velocities of Venus and the Earth are known, as also their proportional distances, it will be easy to compute the time Venus will employ in passing through the arc vw , which, when viewed from the Earth, is equal to the known diameter (or chord) of the Sun CD ; the heliocentric value or length of the arc vw may likewise be readily found. Suppose then an observer at A on the

* Surface.

. K 3

Earth's

Earth's surface to view the planet Venus at v , it will appear just entered within the Sun's disc at c , and passing in the arc vw , will appear to describe the line cd , arriving at d at the end of the computed time. But during this time the observer will, by the Earth's diurnal revolution, be carried from A towards P ; and arriving at P at the same instant that Venus arrives at v , will behold the transit just finishing at d : consequently it will be of a duration proportionally as much shorter than the computed time, as the heliocentric arc vu is shorter than vw . The arc vw is known by computation, therefore, since Venus's motion may in very small arcs be reckoned uniform,

As the computed time
Is to the computed arc vw ,
So is the observed time
To the arc - vu ;

which being taken from vw , leaves the arc uv , that subtends the angle udv . This last angle is the parallax of the base AP ; and the base AP is found by the analogy

As one day or 24 hours
Is to the circumference of the earth (or parallel of latitude)
So is the observed time
To the arc AP , whose chord is the base.

κ But because the minutest errors in a business of this nature are of very great consequence, and because

cause the length of the arc vw , depending on the Sun's diameter, can scarcely be obtained by calculation to that extreme degree of exactness, which is requisite, it is advisable to take another observation on a place so situated on the earth, that the observer being carried in a direction apparently contrary to the former, the errors may counteract each other.

Let the representations be as in the last figure. L If the sun have declination at the time of the transit, B (fig. 52) will represent the pole towards which the sun declines. The observer at A , if at rest, would behold the transit during the time Venus passes from v to w , but being by the Earth's diurnal revolution carried from A through the arc AEF to F , and arriving at F at the instant in which Venus arrives at u , he will perceive the transit just finishing at D ; consequently its duration will be as much longer than the computed time as the heliocentric arc vu is longer than vw . vu being found by the before mentioned analogy, the difference between vu and vw is wu , or the parallax of AF , as before.

Now, in these two cases, a similar error will M have a contrary effect in the first to that which it has in the latter. For if by any error, the computed arc vw (fig. 51) be taken too large, the arc uw , and consequently the parallax will come out too great. But in the latter observation, if the computed arc vw (fig. 52) be taken too large, the arc wu , and consequently the parallax will come out too little. Therefore the mean between

two such observations will be much more to be depended on than either singly.

N By observations on the transits of Venus over the Sun in the years 1761 and 1769, the Sun's mean parallax was found to be $8\frac{1}{2}$ seconds, and hence the Sun's distance is deduced to be very near 1900 diameters of the Earth, or 81818400* geographical miles.

The last three articles in Chap. V, concerning the affections of the planets are deduced from this distance; for,

As the proportional distance of the earth

Is to its real distance,

So is the proportional distance of any other planet

To its real distance.

* A geographical mile is $\frac{1}{60}$ part of a degree of the earth.

C H A P. VII.

OF THE SECONDARY PLANETS.

THE secondary planets, as was before observed, \circ are ten in number, five of which describe orbits about the planet Saturn, four about Jupiter, and one accompanies the Earth. The secondaries of Saturn and Jupiter are observed by the telescope, and by their motions in elongation to the eastward or westward of their primaries is obtained the knowledge of their distances and periodical times, in the same manner as has been already shewn and explained in the planet Venus. Saturn is likewise attended by a phenomenon, which to us appears to be a large broad ring, of no visible thickness. Its breadth is equal to its distance from the body of the planet, and its diameter is to that of Saturn as 9 to 4. The most probable conjecture is, that it consists of a vast number of satellites, which revolve in, and enlighten that region.

Of the two moons of the Georgium Sidus, the p periodical times: First, $8^d\ 17^h\ 1^m\ 19^s$; Second, $13^d\ 11^h\ 5^m\ 1\frac{1}{2}^s$. Distances: First, $33''$; Second, $44''\ 23$.

—Of Saturn's seven moons, the periodical times: First, $1^d\ 21^h\ 19^m$; Second, $2^d\ 17^h\ 41^m$; Third, $4^d\ 13^h\ 47^m$; Fourth, $15^d\ 22^h\ 41^m$; Fifth, $79^d\ 22^h\ 41^m$; Sixth, $1^d\ 8^h\ 53^m\ 9^s$; Seventh, $0^d\ 22^h\ 40\frac{1}{2}^m$.

—Distances in semidiameters of the ring; First, $1\frac{2}{1000}$; Second, $2\frac{1}{100}$; Third, $3\frac{1}{1000}$; Fourth, $8\frac{1}{1000}$; Fifth, $23\frac{1}{1000}$; Sixth, $35\frac{1}{1000}$; Seventh, $27\frac{1}{1000}$.

—Of Jupiter's four moons, the periodical times: q

First,

First, $1^d 18^h 27^m 33^s$; Second, $3^d 13^h 13^m 42^s$; Third, $7^d 3^h 42^m 33^s$; Fourth, $16^d 16^h 32^m 8^s$. Distances in semidiameters of Jupiter: First $5 \frac{667}{1000}$; Second, $9 \frac{177}{1000}$; Third, $14 \frac{384}{1000}$; Fourth, $25 \frac{299}{1000}$.

All the planets, both primary and secondary, receive their light from the Sun. This is evident, because that face only is enlightened which is turned towards that luminary, as may be more particularly seen in our Moon, a greater or less part of which is visible, according to the position in which we lie for viewing the illuminated face. The same varieties are seen in the planets Mars and Venus, not to mention the transits of Venus and Mercury over the Sun, at which time they appear as black unenlightened spots. The phases of Jupiter and Saturn are always round and full, because the Earth is so near the Sun in respect to their distances, that their dark side can never be sensibly turned towards us; yet, that they are opaque, is evident from the disappearing of Jupiter's moons when they enter into its shadow. And though by reason of their vast distance the like obscurations of the satellites of Saturn cannot be observed, yet we can plainly see that the ring casts a shadow on its body: whence we may be certain of the opacity of both: for if the ring were not opaque it could cast no shadow, and if Saturn shone by any native light of his own, the interception of the Sun's light would cause no defect or shadow on his body. It is unnecessary to observe, that the Earth and its Moon are illuminated only on that part or side on which the Sun shines.

When

When one planet intercepts any part of the Sun's s light from another, the planet from which the light is intercepted is said to be eclipsed, if it be a secondary. But if they are both primaries, the inferior planet is said to make a transit. When the moon intercepts the Sun's light from the Earth, it is usual to say the Sun is eclipsed, though, properly speaking, it is the Earth that is eclipsed.

There are three ways in which the satellites of τ Jupiter or Saturn may disappear from an observer placed on the Earth. Thus, let s (fig. 53) represent the Sun, e the Earth in its orbit, j the planet Jupiter and its moons. Then the outermost satellite, for example, will disappear on the enlightened face of Jupiter when at its inferior conjunction m . It will also disappear at its superior conjunction n , being hid behind the body of the planet. And lastly it will disappear when at o , being eclipsed in passing through the shadow of Jupiter.

From these considerations is obtained a good u method of finding the parallax of the Earth's annual orbit. For which purpose the instant of the satellite's first disappearance behind the body of Jupiter must be carefully observed, as likewise the instant of its re-appearance: the middle instant will be that of the superior conjunction at n . In like manner, the middle-instant of the eclipse at o must be found. The time the satellite employs in passing through the arc no will thus be known, and consequently the angle njo . For,

As

As the periodical time of the satellite
 Is to the time of passing the arc NO ,
 So is the whole orbit of 360 degrees
 To the angle NJO .

But the angle NJO is equal to the angle EJS , or
 the annual parallax.

- ✓ By the observations of these eclipses, the discovery
 of the longitude on shore is easily obtained, but the
 violent motion of ships at sea has hitherto prevented
 the use of telescopes on board proper for this purpose,
 though there are good reasons to believe that this
 ✎ difficulty is not insurmountable. From these obser-
 vations it also appears, that light is not propagated
 from luminous bodies in an instant, but passes
 through a given space with an assignable velocity.
 This velocity is extremely great, for it passes through
 the whole distance between the Sun and the Earth
 in about eight minutes; that is to say, at the rate
 of one hundred and seventy thousand miles in a
 second of time. For the periodical times of the
 satellites being known, it is not difficult to deter-
 mine the precise time of any of their eclipses. But
 it is found necessary to make an allowance for the
 position of the Earth with respect to Jupiter, since
 the eclipses happen sooner when the Earth is at r ,
 (fig. 53) in its orbit, than when at a greater dis-
 tance, suppose at z ; and as it is absurd to sup-
 pose, that the position of the Earth should sensibly
 and equally affect the periodical revolutions of
 bodies so vastly remote, and revolving in such dif-
 ferent periods, it is an opinion universally received,
 and

and confirmed by other observations on the light of the fixed stars*, that the eclipses happen later when the Earth is at x than when at F , because the light must in the latter case pass through a space as much greater as the line JE exceeds JF .

C H A P. VIII.

OF THE MOON.

THAT the Moon revolves round the Earth, is x proved from its apparent diameter, which continues at all times, and in all positions, nearly of the same magnitude, whence it may be easily inferred, that its distance from the Earth is nearly at all times the same. Its horizontal parallax, which at a medium is about 57, shews that it is very much nearer to us than the rest of the celestial bodies.

The most remarkable appearance in the Moon is y the continual change of figure to which it is subject. Sometimes it appears perfectly full or circular, at other times half-illuminated, and at other times more or less than half; changing through a very great variety of figures. These changes being always the same at the same elongation from the Sun, are a proof that it receives its light from that luminary: for the Moon is enlightened only on the side that faces the Sun; and a greater or less quantity of that enlightened part is visible to us, according to our

* The aberration of the fixed stars arising from the progressive motion of light, will be explained in Book II. Sect. I.

position. This cannot be better illustrated than by an ivory ball, which being held in the Sun in various positions, will present a greater or less part of its illuminated side to the view of the observer. If it be held nearly in opposition, so that the eye of the observer may be almost immediately between it and the Sun, the greatest part of the enlightened side will be seen. But if it be moved in a circular orbit towards the Sun, the visible enlightened part will gradually decrease, and at last disappear when the ball is held directly towards the Sun. Or, to apply the experiment more immediately to our present purpose; if the ball at any time, when the Sun and Moon are both visible, be held directly between the eye of the observer and the Moon, that part of the ball on which the Sun shines will appear exactly of the same figure as the Moon itself.

- z The Moon's path or orbit is inclined to the plane of the ecliptic, in an angle of about five degrees and
 a a quarter. Its periodical revolution is performed in twenty-seven days, seven hours, forty-three minutes, eleven seconds and a half; but because, during that time the Sun, by its apparent motion, advances considerably in the ecliptic, a space of about two days and a quarter is required by the
 b Moon to overtake it. When the Moon is as nearly in a line between the Earth and the Sun as the inclination of its orbit will allow, it is called the New
 c Moon; and when the Earth is in like manner between the Moon and the Sun, the Moon is said to
 be

be full. The time between two succeeding full moons is called the synodical revolution, and exceeds the periodical revolution, for the reason already given, it being performed in twenty-nine days, twelve hours, forty-four minutes, and three seconds. If the new or full Moon happen near the node, an eclipse takes place; at the new Moon, the Moon being interposed between the Sun and Earth, occasions an eclipse of the Sun; at the full, the Moon entering into the shadow of the Earth, is deprived of the Sun's light, the Earth being interposed between it and the Sun: which phenomenon is called a lunar eclipse, or eclipse of the Moon. At other times, that is, when the new or full Moon happens at a distance from the node, the Moon passes too far to the northward or southward of the ecliptic, either to intercept the Sun's light from the Earth, or to enter the Earth's shadow, and consequently no eclipse happens.

It is determined from observations of angular velocity, parallax and apparent diameter, that the Moon revolves round the Earth in an elliptical orbit, in the focus of which the Earth is placed: and that its velocity is such, that a radius joining its center with that of the Earth does very nearly describe equal areas in equal times.

The line of the apsides, or principal diameter of the Moon's orbit, is not fixed or stationary, but revolves with an irregular or libratory motion from west to east: completing one revolution in almost nine years.

The

- H The line of the nodes is also subject to a like irregular motion from east to west; which is completed in almost nineteen years.
- I The variation of the Moon's motion in any part of its orbit is the difference between its real motion and that which it would have had, provided it had described equal areas in equal times. This is governed chiefly by its elongation from the Sun. During the first quarter its velocity is diminished; in the second quarter, from the quadrature to the opposition or full Moon, it is increased; in the third quarter, from the opposition to the last quadrature, the velocity is again diminished; and from that quadrature to the conjunction, its velocity is again increased. The quantity of angular motion lost exceeds the quantity gained: therefore the whole periodical revolution is performed in a longer time than would have been employed if the Moon were subject to no such variation, but described equal areas in equal times.
- K This variation, and consequently the retardation of the periodical time, is greater when the Earth is in the perihelium, and less when the Earth is in the aphelium: whence it comes to pass, that all the Moon's revolutions are not equal, but are performed in less time in the latter situation than in the former.
- L On all these, as well as other accounts, the determination of the Moon's place in the heavens for a given instant of time has ever been a problem

blem of great difficulty, which till of late years has not been solved to any considerable degree of exactness. Within the last twenty years the commissioners, appointed by the English government for the discovery of the longitude, have particularly attended to this branch of astronomy, and by publishing almanacs in which the Moon's elongation from the Sun, and from certain fixed stars, is ascertained for every three hours, have enabled navigators to determine the situation of ships at sea in general within thirty miles of the truth. This is an advantage of singular use in long voyages, and is at present much used in the royal navy, and East India Company's ships.

CHAP. IX.

CONCERNING THE ECLIPSES OF THE SUN AND MOON.

WE have seen in what manner the periodical revolutions of the celestial bodies, together with the figure, magnitude, and position of their orbits, may be respectively determined by observations made on their apparent motions and situations. From the properties of the ellipsis, and the established law of their velocities (121, K), or otherwise, more immediately from the consideration of gravity (98, w, x), astronomical tables are computed, by which the places of the heavenly bodies may be found for any instant of time. The con-

struction and use of these would lead us too far from the concise and exterior view of phenomena that our limits require: we shall therefore assume, M as a thing granted of course, that the place of any celestial body may be found for any given instant of time.

N The eclipses of the Sun and Moon are phenomena that command the attention even of the vulgar, who have always retained a superstitious veneration for the science of astronomy, chiefly on account of the means it affords of foretelling events of this nature. And though in reality the knowledge required in calculating an eclipse does not essentially differ from that employed in determining the time of the rising and setting of the Sun or Moon, yet there is no doubt but a more particular attention to this subject will be acceptable to the reader.

O As the shadows of the Moon and Earth are the causes of eclipses, it will be necessary first to determine the figure of those shadows. Because the Sun, the Earth, and the Moon are spherical bodies, it follows that the shadows of the two latter must be P either conical or cylindrical; that is to say (fig. 54), if the Sun IK be less than the Earth CD , the shadow of the latter will be part of a cone, whose section is terminated by the lines CE , DF , and whose base is indefinitely distant: or, if the Sun AB be equal to the Earth CD , the shadow will be a cylinder between the lines CG , DH , whose base is indefinitely distant. In either case the shadows of the
Earth

Earth may consequently fall upon and eclipse the superior planets, when in direct opposition to the Sun. But this never happens, and therefore the Sun is neither less than, nor equal to, the Earth, but greater. We know moreover, from the Sun's parallax (136, N), that it is much greater than the Earth, because the Sun's diameter seen from the Earth is about 32 minutes, whereas the Earth's Q diameter seen from the Sun is (126) only about 17 seconds, a quantity that may be regarded as insensible, or inconsiderable in many observations. And since the Sun exceeds the Earth in so high a R proportion, it must of necessity be yet greater with S regard to the Moon, because this last is less than the Earth. Let AB (fig. 55) represent the Sun greater than the Earth CD . The rays of light AC, BD , passing from the extreme edges of the Sun, and in contact with the Earth on the same side, will afterwards meet or cross in the point K . No part of the Sun's light will appear within the cone CKD , which is therefore the shadow in which an observer, being placed, would be totally deprived of the Sun. But there will be a partial shadow or penumbra between those rays ADM, BCL , that pass from the extreme edges of the Sun, and touch the opposite extremes of the Earth: that is to say, an observer between the lines CL and DM , but without the dark cone CKD , will see only a part of the Sun, the rest being hidden by the interposition of the Earth: the quantity of the Sun thus obscured will be greater, and the penumbra darker, the nearer the observer is

placed to the cone $c\kappa d$. Lastly, if the observer be situated beyond the vertex of the dark shadow κ , between the lines κn , κo , formed by the continuation of the extreme rays, he will behold the exterior parts of the Sun forming a lucid ring, environing the Earth on all sides.

- T** The angle $c\kappa d$, at the vertex of the Earth's shadow, is equal to the difference between the diameter of the Sun, seen from the Earth or angle $a c b$, and the diameter of the Earth seen from the Sun, or angle $c b d$ *. Or, if the Earth's apparent diameter from the Sun (147, Q) be rejected as inconsiderable, the angle of the shadow will be equal to the Sun's apparent diameter.
- U** The angle $c i d$, at the vertex of the penumbra, is equal to the sum of the diameter of the Sun seen from the Earth, or angle $a c b$, and the diameter of the Earth seen from the Sun or angle $c a d$: or, if the Earth's apparent diameter from the Sun (147, Q) be rejected as inconsiderable, the angle of the penumbra is equal to the Sun's apparent diameter.
- V** The apparent diameter of any section $e f$, of the shadow, supposed to be viewed from the Earth, namely, the angle $e d f$, is equal to the excess of

* Euclid I. 32. is repeatedly used in what immediately follows: that is to say, in any triangle $a c \kappa$, the outward angle $a c b$ formed by prolonging one of its sides, is equal to the sum of the two inward opposite angles $c \kappa d$, $c b d$: and consequently, that one of the two last-named angles, or $c \kappa d$, will be equal to the difference between the external angle $a c b$ and the other interior opposite angle $c b d$.

the

the Earth's apparent diameter seen from the place of section, namely, the angle CED , beyond the angle at the vertex of the shadow CKD : or, if the angle of the shadow (148, T) be taken as equal to the Sun's apparent diameter, the apparent diameter of any section of the shadow seen from the Earth will be equal to the difference between the apparent diameters of the Sun and Earth, as seen from the place of section, this last diameter being greatest.

The apparent diameter of any section GH , of the w penumbra, supposed to be seen from the Earth, namely, the angle GDN , is equal to the sum of the Earth's apparent diameter seen from the place of section, namely, the angle CND , added to the angle at the vertex of the penumbra CND . Or, if the angle of the penumbra (148, U) be taken as equal to the Sun's apparent diameter, the apparent diameter of any section of the penumbra seen from the Earth will be equal to the sum of the apparent diameters of the Sun and Earth, as seen from the place of section.

Every thing that has been here shewn respecting x the shadows of the Earth is true in like circumstances of the Moon (147, s).

To apply these observations to the facts, let ABY (fig. 56) represent the Sun, CD the Earth, and IK or L the Moon in its orbit KMN ; let the Moon be at IK , between the Sun and Earth; its total shadow may then entirely deprive a part of the Earth at O of the Sun's light, and its penumbra will cause a partial eclipse of the Sun to the inhabi-

Z tants between **G** and **H**. Again, suppose the Moon to be at **L**, and it will itself be eclipsed by the interposition of the Earth between it and the Sun. In lunar eclipses, the Earth's penumbra is not attended to, because its effects in obscuring the Moon cannot be observed with precision by a spectator placed on the Earth.

A It has already been observed (143, E), that eclipses can only happen when the Moon is near one of the nodes of its orbit. Let **ABM** (fig. 57) represent the Sun, viewed from the Earth, **CD** a portion of the ecliptic, or Sun's apparent path, and **EF** a part of the orbit of the Moon; which planet is represented at different times by the circles **G**, **H**, **I**. It is evident, that the eclipse or obscuration of the Sun entirely depends on the position of the node **N**, and the angle of inclination **FND**. If the angle of inclination remain unaltered while the node **N** is very remote from the center **K** of the Sun, the points **K** and **L** may be farther apart than to permit any occultation or apparent contact; and it is clear, that an enlargement of the angle **FND** may produce the same effect: on the contrary, an approach or coincidence of **N** with **K**, or a diminution of the angle **FND** may cause an eclipse, the quantity of obscuration in which will be so much greater, as these circumstances are more prevalent.

B The Sun's place **K** in the ecliptic (146, M) being known from tables, together with the inclination of the Moon's orbit, the place of the node, and of the Moon itself, as likewise the apparent diameters

of the luminaries respectively, it will be easy to find the velocity of the Moon in elongation, and consequently the beginning, middle, end, quantity of obscuration, and other requisites concerning the eclipse. If the computation be made from the tabular places of the heavenly bodies, the result will give the eclipse as seen from the center of the Earth, because, in all tables where the Earth is spoken of, that center is meant, except otherwise mentioned. But it is required to determine the particulars of the eclipse for a given place on the Earth's surface, and this includes the consideration of parallax. The Sun's parallax being very minute (136, N) may in this, and most other cases, be rejected: but the Moon's parallax is so great, that it is at least of as much consequence as any other element whatsoever. For, on this account, the Moon's apparent path, as seen from the surface of the Earth, is so different from that which it would have when beheld from the center, that the same conjunction which gives a total eclipse at one place shall not occasion the smallest obscuration of the Sun when beheld at the same instant from another part of the Earth.

This method of computing a solar eclipse is very operose. For the Moon's parallax at any time past or to come cannot be had without finding its altitude by spherical trigonometry, and other computations must then be made to deduce the apparent positions of the Sun and Moon with their relative velocity, and so forth. And because the altitude of the Moon is continually changing, it is

necessary to repeat the computations of parallax from time to time. To render this business less tedious, it has been found expedient to consider the phenomena of solar eclipses as they would appear to an observer placed on the Moon. Let AB (fig. 58) represent the Earth, seen from the Moon, under an angle of about $1^{\circ} 54'$, or double the Moon's horizontal parallax (141, x). the line CD a portion of the opposite part of the Moon's orbit in which the Earth is seen, the circles G, H, I , shadows of the Moon, that on account of their always being diametrically opposite the Sun, will be found to sense in the ecliptic, especially when the Moon is near the node. The path FE of the shadow will therefore make an angle FND with the line CD , equal to the inclination of the Moon's orbit, and the intersection N will be as far distant from the center of the Earth as the node is heliocentrically from the center of the Moon. Now the motion of the Earth, in the line CD , is equivalent to the Moon's apparent motion in its orbit seen from the Earth, and the motion of the shadow is equivalent to the Sun's motion in the ecliptic. Consequently, the center of the Earth and shadow of the Moon may be projected as seen from the Moon. The diameter of the dark shadow, K, L , or M , seen from the Moon, will be equal to the excess of the Moon's apparent diameter beyond that of the Sun, when both are seen from the Earth (148, v. 149, x), the Moon's apparent diameter being greatest, but if
it

it be the less of the two, the shadow will not reach the Earth. The diameter of the penumbra, *G, H*, or *I*, seen from the Moon, will be equal to the sum of the apparent diameters of the Sun and Moon, seen from the Earth (149, *w, x*). With these data the eclipse may be constructed universally.

But in constructing the eclipse for a particular *G* place, the rotation of the Earth on its axis must be brought into consideration. For while the shadow passes over the Earth's disc, a given place *P* will be carried round in its parallel of latitude, and may likewise be marked in the projection for any instant of time. When the place enters the penumbra, the eclipse will begin there; when the place and the center of the shadow are the nearest, the obscuration will be greatest, and when the penumbra leaves the place, the eclipse will end. If there be a dark shadow, and it passes over the place, the eclipse will be total. If there be no dark shadow, and the place should pass within the penumbra to a depth exceeding the Moon's apparent diameter, the Sun will be seen environing the Moon on all sides; whence the eclipse is said to be annular (148, *s*). And, in general, in any solar eclipse that is not annular, the distance of the place within the penumbra will measure the greatest section or part of a diameter of the Sun obscured at that instant, and the line joining the cusps, or angular termination of the apparent part of the Sun, will be at right angles to the measuring line or diameter,

meter, of which the measuring line represents a part.

¶H We are now to consider an eclipse of the Moon. It is evident, that the difference in the phenomena of a solar eclipse would not take place if the parallax of each luminary were the same; because, whatever mutation of place the parallax might occasion in the one, the same would be produced in the other, and they would neither approach nor recede from each other on that account. Now the section of the Earth's shadow passed through by the Moon in a lunar eclipse, being at the same distance from the Earth as the Moon itself, must be subject to the same parallax at equal altitudes; and since the individual points of immersion, emersion, or other periods of the eclipse must in the shadow have the same altitudes the parts of the Moon they, as it were, lie on and obscure, the effects of parallax must be the same on both. Rejecting therefore the consideration of parallax, the Earth's shadow *AB* (fig. 59) may be taken to occupy a place in the heavens diametrically opposite the Sun, and having an equal and similar motion to the apparent motion of that luminary, its apparent diameter, seen from the Earth, will be equal to the difference between the apparent diameters of the Earth and Sun, as seen from the Moon (148, v). Or it will be equal to twice the horizontal parallax of the Moon diminished by the subtraction of the Sun's apparent diameter. And if the inclination of the orbit of the Moon be found, there will be a certain distance

distance of the node N from the center of the shadow C , that will require the Moon near the opposition to pass through the Earth's shadow, and be consequently eclipsed. From the greater or less distance of the node N , or M , it will be determined whether the eclipse will be partial or total; and from the respective places, the quantity and direction of the relative velocity, together with the apparent magnitudes of the shadow and the Moon, all the particulars of the eclipse may be known without difficulty.

It may with great reason be demanded, how it happens that the Moon, which is affirmed to emit no light of itself, but only by reflection of the Sun, is nevertheless sufficiently luminous, even in the very middle of a total eclipse, to be distinctly seen of a dusky reddish color. The Earth's atmosphere, or body of air that surrounds it, is the cause of this phenomenon. In fact, the shadow of the Earth itself never extends so far as the Moon's orbit, though the shadow occasioned by the dispersion or reflection of the light, that falls on the atmosphere may, with a very small allowance, be taken for the shadow which the Earth would have had if the light had passed close by it without interruption. We cannot with regularity explain the refraction of light in this place. It will therefore be sufficient to observe, that in the event now under consideration the Sun's light falling obliquely on the atmosphere, is bent or turned out of its course so as to converge sooner to a point, than it would otherwise

otherwise have done; the spherical atmosphere performing, in some measure, the office of a large convex lens, or burning glass. The more obliquely the rays fall, the greater is their deviation from their original course; and those rays that pass close to the Earth are found, by observations on the setting Sun and other heavenly bodies, to suffer a refraction of about 33 minutes of measure. The laws of optics, hereafter to be explained, require that they should undergo an equal refraction in passing out through the opposite part of the atmosphere. Each exterior ray of the real shadow will therefore pass 66 minutes within the rays that would have formed the cone CKD , fig. 55. and consequently, the angle at the vertex of the cone will be 132 minutes, or $2^{\circ}.12$ greater than it would have been; that is, it will be equal to the diameter of the Sun $32^{\circ}.148, 7$, added to $2^{\circ}.12$, which gives $2^{\circ}.44'$. Hence the axis of the cone, or length of the shadow, is found to be no more than 42 semidiameters of the Earth; whereas the radius of the Moon's orbit, or mean distance of the Moon, is about 60 semidiameters of the Earth. In the space between the penumbra and the Earth's real shadow it is much darker than the penumbra, though that space is illuminated by the rays of the Sun, which are variously refracted, according to the density of the air they pass through. Many rays are reflected back, and the rays that go forward are such whose nature does not admit of their being easily reflected. We are to shew in
future

future that these are the red, orange, and yellow. Hence it is that the Moon in an eclipse appears red; and a spectator on the Moon would, after losing sight of the Sun, behold the Earth environed with a narrow luminous edge of bright red light, shaded off with yellow on the outside.

Since the Earth, when beheld from the Moon, must always appear in the part of the heavens immediately opposite the Moon's apparent place as seen from the Earth, the enlightened side of the Earth will have the same figure, when seen from the Moon, as the dark side of the Moon would exhibit if it could be seen at the same instant from the Earth. Thus, when the Moon is invisible, or near the conjunction, the Earth is in opposition, and presents a full luminous face to the Moon; and on the contrary, when the Moon is at the full, or opposite the Sun, it must be on the dark side of the Earth, which consequently then becomes invisible. Near the beginning of the first or end of the last quarter, the dark side of the Moon is rendered visible by the full Earth shining on it, but is scarcely so luminous as the Moon when eclipsed. The Earth's disc, seen from the Moon, is about thirteen times that of the Moon seen from the Earth. If the Earth reflected as great a part of the light that falls on it as the Moon does, its light at the Moon would exceed the moon-light with us in that ratio. This, however, is not probable, though it may fairly be supposed that it is three or four times as great. But

we

we have already observed that the Earth's atmosphere, in a lunar eclipse, illuminates the Moon rather more than this. Whence it follows, that the narrow ring of light encircling the Earth, when seen from the Moon during an eclipse, gives a light far exceeding our moon-light at the full, or even that of the Earth's full face shining on the Moon: and as the surface of the ring, by computation, can hardly equal the one hundredth part of the Earth's disk, or the one eighth part of the Moon's disk, its brightness will be more than twenty-four times that of the Moon. It must consequently be very luminous and dazzling.

Notwithstanding this, there have been eclipses of the Moon, when in that part of its orbit near the Earth, in which that luminary entirely disappeared. But these observations are very rare.

C H A P. X.

OF COMETS; AND OF THE PROPORTION OF LIGHT
AND HEAT ON THE PLANETS.

BESIDES the seven primary planets already enumerated, and their moons or attendants, there are other bodies that revolve round the Sun, and claim peculiar distinction on several accounts. These are called Comets, and appear occasionally in every part of the heavens; their motions being performed in very long ellipses, whose lower focus is in or near the Sun. By observations of parallax it is found, that at their first appearance they are nearer to us than Jupiter; whence it is concluded, that they are most commonly less than that planet; for if they were as large as Saturn, they would be seen as far off.

When a comet arrives within a certain distance of the Sun, it emits a fume or vapour, which is called its tail. This shews that they contain a portion of matter considerably more rare and volatile than any on the Earth; for the tail begins to appear while they are yet in a higher, and consequently colder region than Mars. The tail is always directed to that part of the heavens which is directly

directly or nearly opposite to the Sun; and is greater after the comet has past its perihelium, than during its approach towards it; being greatest of all at the time when it has just past the perihelium.

- v That part of a comet's orbit which comes under our observation is so small in proportion to the whole, that in most it does not differ from a parabola, by quantities that observation can distinguish: for which reason the dimensions of their orbits and periodical times cannot be determined with any degree of precision from a single appearance. But from the re-appearance of comets after long intervals of time in the same region of the heavens, and moving in the same curve, it is decided that they revolve about the Sun in very long or eccentric ellipses; being governed throughout by the same law of describing equal areas in equal times, which is found to take place in the inferior part of their orbits. The comet that appeared in the year 1661 was seen before in the same orbit, and under the same circumstances in the year 1532: which shews its period to be 129 years. So likewise, the comet that appeared in the years 1456, 1531, 1607, 1682, and 1759, is determined to revolve in a period of about seventy-six years. And that very remarkable comet which was observed in the year 1680, is shewn to be the same with that which appeared in the year 1106; its period being 575 years.

The

The number of comets is very much greater w than that of the planets which move in the vicinity of the Sun. From the reports of former historians, as well as from the observations of late years, it is ascertained, that more than four hundred and fifty have been seen previous to the year 1771: and when the attention of astronomers was called to this object by the expectation of the return of the comet of 1759, no fewer than seven were observed in the course of seven years. From this circumstance, and the probability that all the comets recorded in ancient authors were of considerable apparent magnitude, while the smaller were overlooked, it is reasonable to conclude, that the number of comets is considerably beyond any estimation that might be made from the observations we now possess. But the number of comets whose x orbits are settled with sufficient accuracy to ascertain their identity when they may appear again is no more than fifty-nine, reckoning as late as the year 1771. The orbits of most of these are in- y clined to the plane of the ecliptic in large angles, and the greater number of them approached nearer the Sun than the Earth ever does. Their motions z in the heavens are not all in the order of the signs, or direct, like those of the planets: but the number whose motion is retrograde is nearly equal to that of those whose motion is direct.

It is not necessary in this work, to enter fully A into the consideration of final causes; more particularly as the subject has embarrassed the greatest

metaphysicians, and may with justice be said to be too extended for the human powers. In every thing we see, the phenomena considered singly, are necessary consequences of certain general laws, to which the universe appears to be subjected, but when they are considered in a collective view, a certain relation, or fitness for producing some general effects, is seen, which by no means depends on the same laws, and by analogy is referred to the operation of an intelligent agent. To illustrate this by an example, of the simplest kind, we may observe, that in the well-known instrument called scissars, it follows necessarily from the laws of motion, already explained in the mechanical powers (60, G. 67, H), that the blades will cut or divide certain substances exposed to their action; but if we consider the various circumstances that co-operate in producing this effect, we must disclaim all reasoning from analogy before we can resolve their connection into an effect of those laws, without supposing the agency of an intelligent being as the cause of their union, and concluding that it was intended they should jointly concur in one purpose. It is to this being that we refer, in order to decide, why the sharp edges were made on the inner, rather than on the outer, part of each blade: why the other extremities have annular terminations: why the instrument is made of steel rather than lead; and so forth. The purposes or motives which determine the actions of intelligent beings, and produce their effects in a manner similar

lar to the operation of the laws of nature or properties of matter, in cases where thought is not supposed to be concerned, are called final causes. In the works of nature we behold enough of exquisite contrivance, and can see far enough into many final causes to convince us that the arrangement of the universe has been made, and probably is still occasionally adjusted, by a being whose intelligence and power is immensely beyond that which we possess. To judge properly of his intentions, or in other words, to be equal to the task of exploring the science of final causes, requires no less than a perfect knowledge and recollection of every purpose to which the objects around us may be applied, together with a clear conception of the ideas of fitness and order that form the prototypes in the mind of that great being who directs their motions. These considerations shew the absurdity of attempting to explain the final causes of every event we see, but they by no means require that we should neglect them in cases where we have reason to believe we understand the phenomena, and have sufficient experience to be assured that we discern the principal, or at least one of the principal purposes to which things may have been destined. Thus, it is scarcely to be imagined that we can err in concluding, that the eyes, ears, legs, wings, and other parts of animals were made for the purposes of seeing, hearing, walking, flying, and the like. Neither can we avoid inferring, that the power who constructed living creatures with

mouths, teeth, and organs to digest and assimilate food for their nutriment, did likewise form other organized bodies, which we call vegetables, for the express purpose of affording that food. It is needless to multiply instances. We cannot avoid seeing them every moment, and their effect is so striking, that we are insensibly forced from analogy to allow the existence of a final cause in all cases, whether we are able to discover it or no.

On this ground, an enquiry into the final causes of the planetary bodies offers itself to our consideration. The earth is shewn to be a planet in circumstances very similar to the other five: we know its final cause—to support a number of inhabitants. And by analogy, we may conclude, that the others are also habitable worlds; though from their different proportions of heat it is credible, that beings of our make and temperature could not live upon them. However, even that can scarcely be affirmed of all the planets: for the warmest climate on the planet Mars is not colder than many parts of Norway or Lapland are in the spring or autumn. Jupiter, Saturn, and the Georgium Sidus, it must be granted, are colder than any of the inhabited parts of our globe. The greatest heat on the planet Venus, exceeds the heat in the island of St. Thomas on the coast of Guinea, or Sumatra, in the East Indies, about as much as the heat in those places exceeds that of the Orkney islands, or that of the city of Stockholm in Sweden: therefore,

fore, at 60 degrees north latitude on that planet, if its axis were perpendicular to the plane of its orbit, the heat would not exceed the greatest heat of the Earth, and of course, vegetation like ours might be there carried on, and animals of the species on Earth might subsist. If Mercury's axis be supposed to have a like position, a circle round each pole of about 20 degrees diameter would enjoy the same temperature as the warmer regions of the Earth, though in its hottest climate water would continually boil, and most volatile compounds would be parched up, destroyed or dissipated into vapor. But it is not at all necessary that the planets should be peopled with animals like those on the Earth: the Creator has doubtless adapted the inhabitants of each to their situation.

From the observations that have just been made, a better notion may be formed of the proportions of heat on the planets than can be conveyed by numbers. It will not however be remote from our purpose to compare the light of the superior planets with that of our day, from whence it will appear, that they are by no means in a state of darkness, notwithstanding their great distance from the Sun. This might be instanced by several different methods, as by the Sun's light admitted into a dark chamber, and received on paper with different degrees of obliquity; by a greater or less number of candles brought into a room for the purpose of illuminating it with different proportions of light; or by various optical methods that need not be

here described. It will be sufficient for the illustration of the subject to compare their different proportions of light with that of a moonshine night at the time of the full.

- G When the Moon is visible in the day-time, its light is so nearly equal to that of the lighter thin clouds, that it is with difficulty distinguished amongst them. Its light continues the same during the night; but the absence of the Sun, suffering the aperture or pupil of the eye to dilate itself, renders it more conspicuous. It therefore follows, that if every part of the sky were equally luminous with the Moon's disc, the light would be the same as if in the day-time, it were covered with the thin clouds above-mentioned. This day-light is consequently in proportion to that of the Moon, as the whole surface of the sky, or visible hemisphere, is to the surface of the Moon; that is to say, nearly as 90,000 to 1. The light of the Georgium Sidus being to that of the Earth as 0.276 to 100, will be equal to the effect of 243 full Moons. The light of Saturn will be equal to that of 990 full Moons: Jupiter's day will equal the light of 3330 Moons, and that of Mars will require 38,700, a number so great, that they would almost touch one another. It is even probable, that the Comets, in the most distant parts of their orbits, enjoy a degree of light much exceeding moonshine.
- 1 If the Comets be habitable they must be possessed by creatures very different from any we have been used to behold and consider. There may, however,

however, be other uses for which it is probable they may have been formed: the matter that composes their tails must fall in process of time to the Sun or the nearest planet that may pass through it, where it may supply defects, and answer purposes which our total ignorance of its properties scarcely allows us even to conjecture. In the Sun it may serve to recruit the waste of matter that luminary may suffer by the constant emission of the particles of light. After a great number of revolutions, the resistance of the Sun's atmosphere, and a concurrence of circumstances, may occasion the comet itself to approach the Sun, and at length fall into it, and become a part of its body.

C H A P. XI.

OF THE TELESCOPIC APPEARANCE OF THE MOON.

THE observations which might confirm the hypothesis of planetary worlds seem to be placed beyond our power. We can scarcely hope to make optical instruments sufficiently perfect to render their inhabitants visible to us. The gross air that furrounds us, is a great impediment to the use of those we already possess, and limits their perfection to a certain degree, beyond which we cannot pass. All, therefore, that we can do, is to examine if the planets are accommodated with those things which we are used to consider as

necessary to animal existence. Lands, seas, clouds, vapours, and an atmosphere or body of air, are objects that we may expect to find on the face of a habitable world: it is our present business to relate what has been done in this respect.

- L** The Moon being so very near us, and likewise in the same temperature as to light and heat, offers itself as the fittest body for examination. We discern a variety of spots with the naked eye, which the imagination naturally supposes to be seas, continents, and the like; but on a more accurate inspection, with the assistance of the telescope, it is perceived that many of those appearances are occasioned by vast obscure pits or cavities, and elevations or mountains. The heights of these mountains may easily be found; for by the horizontal parallax we know that the Earth's apparent diameter, seen from the Moon at its mean distance, is $1^{\circ} 54'$ ($132, \times$) or 6840 seconds, while that of the Moon seen from the Earth at the same distance **M** is $31' 29''$, or 1889 seconds. Their absolute diameters must therefore be in proportion to these numbers. Consequently, if we find the proportion the height of a lunar mountain bears to the Moon's diameter, we may, without difficulty, find the quantity of that height in miles or other terrestrial dimensions.
- N** These mountains and cavities are known to be such from their shadows. In the first and second quarters, when the Sun shines obliquely on the face
of

of the Moon, the elevated parts cast a triangular shadow in the direction from the Sun; and, on the contrary, the cavities are dark on the side next the Sun, and illuminated on the opposite side. The shadows shorten as the Sun becomes more directly opposed to the anterior face of the Moon, and at length disappear at the time of the full. During the third and last quarters, the shadows appear again, but all fall towards the contrary side of the Moon, though still with the same distinction, namely, that the mountains are dark and shady on the side furthest from the Sun, and the pits are dark on the side next the Sun. The same deduction is obtained by contemplating the inner illuminated edge of the Moon. If the Moon were a perfect sphere, this edge would be a regular curve, but if its surface be diversified by hills and cavities, it is evident that the higher parts must be enlightened sooner, and the cavities later than the rest of the surface. This is accordingly the case, and affords a method of obtaining the heights of the mountains.

To render the explanation easier, we shall suppose the Moon to be in its quadrature, and the mountain to be situated near the Moon's apparent center.

Let the circle $ABDC$ (fig. 61) represent the Moon, whose center is c ; and E the Earth: then a spectator at E will see the Moon enlightened in the half AGD , and the line EC will pass through A , or the inner enlightened edge. The ray of light SAB touching the Moon at A , will cross the line EC at
right

right angles, and illuminate the top of the mountain B. The angle AEB is found by observation, then in the triangle AEB,

As the co-sine of the observed angle	-	AEB
Is to the Moon's distance	-	AE,
So is the sine of the observed angle	-	AEB,
To the side or line	-	AB.

Then in the right angled triangle CAB, the sides CA and AB being known, the side CB is found from the well known property (Euclid I. 47); that is to say, the square of the Moon's semidiameter CA being added to the square of the line AB, the square root of the sum is the side CB. And if the semidiameter of the Moon CF be taken from the line CB, the remainder is FB, or the height of the mountain.

P From observations of this kind,* it appears that the lunar mountains are much higher in proportion to its radius than any we have upon the Earth. Herschel has observed several volcanos in these mountains.

Q That the Moon is surrounded by an atmosphere or body of air, is rendered probable by many observations of solar eclipses, in which the limb or edge of the Sun was observed to tremble just before the beginning. The planets likewise are observed to change their figure from round to oval just before the beginning of an occultation behind the Moon; which can be attributed to no other cause, than that their light is refracted, being seen through the Moon's atmosphere. Many astronomers are of opinion, that the Moon has no atmosphere,

sphere, because we see no clouds, and because the fixed stars disappear at once at the time of an occultation without any gradual diminution of light, which they suppose ought to take place. But if we consider the effect of days and nights near thirty times as long as with us, we may readily grant that the phenomena of vapours and meteors may be very different: perhaps their clouds and rain, if any, may be condensed into visible quantities, only during the absence of the Sun, and if so, it is no wonder that we never see them. With respect to the fixed stars, it is plain, that granting the Moon to have an atmosphere of the same nature and quantity as ours, no such effect as a gradual diminution of light ought to take place, at least as to sense. Our atmosphere is found to be so rare at the height of 44 miles, as to be incapable of acting on the rays of light. This height is the 180th part of the Earth's diameter; but since clouds are never observed higher than four miles, we must conclude, that the vaporous or obscure part is but the 1980th part. The mean apparent diameter of the Moon is $31' 29''$, or 1889 seconds; therefore, the obscure part of its atmosphere, when viewed from the Earth, must subtend an angle of less than one second, which space is passed over by the Moon in less than two seconds of time; a space and time so short, that it can hardly be expected that observation can in general determine whether the supposed obscuration takes place or not.

It

- R** It must, however, be allowed, that the Moon's atmosphere, if it has one, is certainly much less considerable than that of the Earth. For there is seldom any appearance, in an eclipse of the Sun, of the remarkable phenomenon (158, Q), that the Earth's atmosphere is shewn to produce in a lunar eclipse*.
- S** The Moon turns round on its own axis once in the time of its periodical revolution. This is evident, because the same face or side is constantly turned towards us. For a spectator on the Moon will see the Earth carried through every part of the ecliptic in the course of one revolution; and as the same face of the Moon is constantly turned towards the Earth, it must be successively turned to every part of the ecliptic to which the Earth apparently moves. But if it be successively turned to every part of a great circle in the heavens, it must revolve on its axis. By this slow rotation, it appears, that the inhabitants of the Moon have but one day and night in the course of a month.
- T** This rotation on its axis is the most uniform motion the Moon has; but its uniformity occasions a seeming irregularity, which is termed the libration. For as the Moon's motion in its orbit was shewn to be not uniform (143, F) the effect it has in turning its face from the Earth is likewise subject to the same irregularities; for instance, in the swiftest part of the revolution, its motion in its orbit turns its face

* See Philosophical Transactions, Anno, 1779, for an account of the solar eclipse of June 24, 1778, in which this appearance was seen by Don Antonio Ulloa.

from the Earth something more than the rotation on its axis turns it the other way, and therefore it appears to have a small motion on its axis towards the east, by which some of the more western parts are brought to view, and an equal portion of the eastern limb disappears. In the slower part the contrary is seen, for then the rotation on its axis prevailing, brings the western parts into view, and the eastern disappear. This is called libration in longitude.

There is another kind of libration that arises from the Moon's axis being inclined to the plane of its orbit, by which means sometimes one of its poles, and sometimes the other, is inclined towards the Earth. In consequence of this, we see more or less of the polar regions at different times. This is called libration in latitude.

C H A P. XII.

OF THE TELESCOPIC APPEARANCES OF THE SUN
AND THE PLANETS.

V THE Sun is not without spots on its disc, but they are seldom so large as to be seen by the naked eye. When viewed with a telescope, the eye being defended by a piece of coloured or smoked glass, they are found to appear in various forms and numbers. The larger spots, most of which exceed the whole Earth in apparent magnitude, last a considerable time; sometimes three months before they disappear, at which time they are generally converted into faculæ, or spots which exceed the rest of the Sun in brightness. They are of no constant figure, frequently changing during the time of observation, and sometimes one dividing into several smaller ones. In general they consist of a nucleus or central part, much darker than the rest, which is surrounded by a mistiness or smoke. The general opinion concerning them is, that they are occasioned by the smoke and opaque matter thrown out by volcanos or burning mountains of immense magnitude; and that when the eruption is nearly ended, and the smoke dissipated, the fierce flames are exposed and appear as faculæ, or luminous spots. But Dr. Alexander Wilson of Glasgow has established from observation, that most, if not all the spots, are excavations in the luminous matter that

that environs the body of the Sun, probably to no great depth*. At present (anno 1779) they are often seen to the number of thirty or more, but there have been periods of more than seven years, in which none have been observed.

All the spots of the Sun have an apparent motion w from east to west, which is quicker when they are near the central regions than when near the limb. This proves that the sun revolves on its axis from x west to east, and likewise that its figure is spherical. The period, as observed by Cassini, is 25 days, 14 r hours, 8 minutes. From the line of the motion of the spots, which is sometimes strait, but oftener curved or elliptical, it is discovered that its axis is z not perpendicular to the ecliptic, but inclined, so as to make an angle with the perpendicular of about seven degrees and a half.

The zodiacal light is attributed to the solar atmo- a sphere. This remarkable phenomenon accompanies the Sun. When it begins to appear before sun-rise, it seems at first sight like a faint, and almost imperceptible, whitish light, resembling the milky-way, and ill-terminated, which is almost confounded with the twilight that is seen commencing near the horizon. It is then little elevated, and its termination may sometimes be discerned in a conic or conoidal form. In short, its figure agrees with that of a very flat or lenticular spheroid seen in profile. As it gradually rises above the horizon it becomes brighter and larger to a certain point, that may be call-

* Philosophical Transactions, 1774-

ed its maximum, after which the approach of day renders it gradually less apparent, and at last invisible. The direction of its longer apparent axis is observed to be in the plane of the Sun's equator; but its length is subject to great variation, so that the distance of its summit from the Sun varies from 45° to even 120° . These great differences in magnitude and brightness may perhaps depend considerably on some, yet unknown, circumstances in our atmosphere. It is usually seen to the greatest advantage about the solstice.

B It has been supposed, that this atmosphere is the cause of the ascent of the vapor which forms the tails of the comets, and which is always carried to that part of the heavens which is opposite the Sun. But the direction of these vapors may perhaps be determined by the action of the particles of light emitted from the Sun.

C The planet Mercury is at all times so near the Sun, that we can only distinguish with the telescope a variation in its figure, which is sometimes that of a half Moon, and sometimes a little more or less than half. Whence it follows, that its form is globular, and that it receives all its light from the Sun.

D The planet Venus, when viewed through the telescope, has a very pleasing appearance. At the time of its greatest elongation it appears like the Moon in the quadratures; one half of its disc being enlightened. In the inferior part of its orbit, as its elongation decreases, the enlightened part becomes less,

less, appearing falcate, or horned; after passing the inferior conjunction, the planet is again seen horned, but the illuminated part then increases, and at the greatest elongation, half is again seen enlightened. In the superior part of its orbit, as its elongation decreases, its face becomes more full and round, till the superior conjunction, after which time it is again diminished by the same gradation as its increase was in the former case accomplished. There is no difficulty in accounting for this variety of phases, it being occasioned by the different positions of Venus with respect to the Sun and Earth: for if the enlightened face of Venus must of course be always opposite to or facing the Sun, it will be more or less visible to us according to our situation at various times.

The surface of Venus is diversified with spots like our Moon, by the motion of which it is determined, that it revolves on its axis from west to east in the space of twenty-three hours. When the air is in a good state for this kind of observations, mountains like those in the Moon may be discerned, with a very powerful telescope. Late observations of Herschel have however rendered these accounts uncertain*.

The face of the planet Mars is always round and full, as its superior situation requires, excepting at the time of the quadrature, or elongation of 90 degrees, when a small part of the unenlightened hemi-

* Philosophical Transactions. 1793.

sphere being turned towards us, its disc appears like the Moon about three days after the full.

- G By the spots on Mars, its diurnal revolution is ascertained in the direction from west to east. From the ruddy and obscure appearance of this planet, as well as from other appearances, it is concluded, that its atmosphere is nearly of the same density as that of the Earth. Mr. Herschel has observed, that two circles surrounding the poles of this planet are very white, and luminous, probably, from snow lying there.
- H We have already had occasion to speak of the satellites of Jupiter and Saturn. The annual parallax of these planets is not considerable enough to bring any sensible part of their dark hemispheres towards us in any position of elongation; consequently their faces are always round and full.
- I The telescopic appearance of Jupiter affords a vast field for the curious enquirer. It is in general encircled with one or more obscure belts or bands parallel to the plane of its orbit, and consequently to each other. These are not regular or constant in their appearance. They have been seen to the number of five, and during the time of observation two have gradually disappeared. Sometimes but one is seen; and sometimes, when the number is more considerable, one or more dark spots are formed between the belts, which increase till the whole is united in one large dusky band. The spots of Jupiter are the brighter parts of its surface, and are not permanent, though more so than

than the belts; yet it is found that they re-appear after certain unequal intervals of time. The remarkable spot, by whose motion the rotation of Jupiter on its axis was determined, disappeared in 1694, and was not seen again till 1708, when it re-appeared exactly in the same place on its surface, and has been occasionally seen ever since.

It has been conjectured, that these belts are seas, and that the variations observed both in them and the spots are occasioned by tides, which are differently affected, according to the positions of his moons. It is probable, however, that they are in its atmosphere. If an observer, possessed of skill and patience equal to the task, would delineate the phases of Jupiter for the space of a periodical revolution, noting at the same time the positions of his satellites, this opinion might be either established or rejected: but at all events such a series of observations could not fail to throw great light on the subject.

The very great distance of the planet Saturn, and the tenuity of its light, do not permit common observers to distinguish those varieties which it is probable are on its surface. Herschel's telescopes shew belts on its surface. These are generally parallel to the ring. Saturn is found to revolve on an axis perpendicular to the plane of the ring, and its diameter is shorter than that which is measured in the plane of the ring in the proportion of 10 to 11. The ring itself is inclined to the ecliptic; in consequence of which, its ap-

parent figure is continually varying. When the line of its nodes points directly towards the Earth, the ring, presenting its edge to the observer, becomes invisible to common telescopes: if the same line points directly towards the Sun, the ring becomes invisible for want of illumination: and lastly, if the plane of the ring passes between the Sun and the Earth, the ring cannot then be seen, because its dark side is towards us. At all other times its figure is that of an oval, which is broader or narrower accordingly as the line of the nodes is farther from or nearer to the above positions.

C H A P. XIII.

OF THE LENGTH OF DAYS AND NIGHTS; AND OF THE SEASONS.

M WE have seen that every planet which is accessible to observation has a revolution on its axis; the intention of which is, undoubtedly, to give alternate night and day to every part of their surfaces. An inclination of the axis of any planet to the axis of its orbit, by causing the length of days and the intensity of heat to vary, will occasion a vicissitude of seasons. On this account Jupiter, whose axis is nearly perpendicular to the plane of its orbit, has equal days and nights on every part of its surface at the same time, the days being four hours and twenty-eight minutes, and the nights of the same length. But the planets Mars and Venus, whose axes are
inclined

inclined to the planes of their respective orbits, have each an annual change of seasons and length of days. The Earth, for the same reason, has a similar vicissitude, the explanation of which will render it unnecessary to enlarge on the circumstances of the other planets.

For this purpose it will be useful to define those N imaginary circles, which astronomers and geographers have invented for the purposes of methodizing and facilitating the communication of science.

On the Earth, a great circle, supposed to be O drawn at an equal distance from each pole, is termed the Equator: less circles drawn parallel to the equator are called Parallels of Latitude; and great circles intersecting the equator at right angles, and consequently passing through the poles, are called Meridians. But when the meridian of a place is spoken of, it is usually understood to be a semicircle passing through the given place, and terminating at the poles. The other half which completes that whole circle, is then called the opposite meridian.

In the heavens, a great circle, parallel to the P equator, is termed the Celestial Equator; but the less circles parallel to it are called Parallels of Declination; and the great circles intersecting it at right angles, and passing through the celestial poles of the Earth, are called Hour Circles, or circles of right ascension.

The ecliptic is that great circle in the heavens, Q in which the Sun describes its apparent annual

course: less circles, drawn parallel to the ecliptic, are called Parallels of Latitude; and great circles intersecting it at right angles, and consequently passing through its poles, are called Celestial Meridians.

R The horizon is that great circle which divides the visible or upper hemisphere from the lower. If this circle have the eye of the observer for its center, it is called the Sensible Horizon; but if its center be that of the Earth, it is termed the Rational Horizon. To this last all astronomical observations are reduced or applied; the former being only considered as one of the parallels of altitude. Less circles, parallel to the horizon, are called Parallels of Altitude, if above, but of Depression, if below the horizon, and the great circles intersecting it at right angles, are called Azimuths.

S The point of the heavens, which is immediately above the observer, or is elevated 90° above the horizon, is termed the Zenith; the opposite point immediately beneath, or at 90° of depression below the horizon, is termed the Nadir.

T Latitude on the Earth is an arc of the meridian, contained between a given place and the equator. It is measured in degrees and minutes of the meridian. In the heavens it is an arc of the celestial meridian, contained between a given place and the ecliptic.

U Longitude on the Earth is an arc of the equator, contained between the meridian passing through a given place and the first meridian. It never exceeds a semicircle. The first meridian on the Earth is

is arbitrary; but the English astronomers in general reckon from that which passes through the observatory at Greenwich. Longitude in the heavens is an arc of the ecliptic, contained between a given meridian and that which passes through the first point of the constellation Aries; the said point being always at the western extremity of the arc.

Right ascension is an arc of the celestial equator, contained between a given hour circle and that which passes through the first point of the constellation Aries; the said point being always at the western extremity of the arc. Declination is an arc of the hour circle, contained between a given place and the equator.

The circle which divides the enlightened hemisphere of a planet from its dark hemisphere is called the Terminator. It may in most cases be considered as a great circle.

Let $NESQ$ (fig. 62 and 63) represent the Globe x of the Earth, and c the Sun: then the circles NMS , NMS , &c. will represent the meridians intersecting the Equator EQ at right angles, and passing through the poles N and S . The lines pp , pp , &c. will represent the parallels of latitude; and the line CM will represent the plane of the Earth's orbit.

Now it is evident, that it is day at any given place y on the globe, so long as that place continues in the enlightened hemisphere; and that when by the diurnal rotation it is carried into the dark hemisphere it becomes night; twilight not being here considered. And from the contemplation of figure

63, it appears, that if the poles be situated in the terminator, the terminator will divide each of the parallels into two equal parts, and consequently, since the uniform motion of the Earth causes any given place to describe equal parts of its parallel in equal times, the days and nights will be equal on every parallel of latitude; that is to say, all over the globe, except at the poles, where the Sun will neither rise nor set, but continue in the horizon.

- z But if, as in figure 62, the axis be not placed in the plane of the terminator, the terminator will divide the equator into two equal parts; but the parallels which are situated towards the enlightened pole will have a greater part of their peripheries in the enlightened than in the dark hemisphere: while similar parallels towards the other pole will have a like greater part of their peripheries in the dark hemisphere. Whence it follows, that the first-mentioned parallels will enjoy longer days than nights, and the contrary will happen to the latter, they having shorter days and longer nights; while at the equator the days and nights continue equal. All this is evident from the figure, where it is also observable, that the disproportion is greatest in the greater latitudes; and that places, whose distance from the pole is less than that of the pole from the terminator, must enjoy either a constant day or constant night, the rotation of the Earth never carrying them into the opposite hemisphere.

In this position of the axis the inhabitants on the one side of the equator may be said to enjoy summer,

mer, and those on the other side winter with respect to each other. For the long duration of the Sun above the horizon must occasion a proportionally greater degree of heat, and its longer absence from places situated in the other hemisphere must have the contrary effect.

But this is not the only cause of the difference of heat at the different places. A spectator at G , which is 90° distant from the terminator, will have the Sun in the zenith; a spectator at T will see the Sun in the horizon; and, for every intermediate distance, the arc of a great circle comprehended between the terminator and the place of observation will be the measure of the Sun's altitude. Therefore every parallel between Q and the enlightened pole will have the meridian altitude of the Sun increased, by the angle NMT , beyond what it would have been had the pole continued in the plane of the terminator: and every place between Q and the dark pole will have the Sun's meridian altitude diminished by the same quantity. Between G and Q the former altitude thus increased, making a sum greater than 90° , the altitude must be measured by its complement. And between S and T the altitude will be a negative quantity, or beneath the horizon. This difference of the altitudes of the Sun must cause an increase of heat towards the enlightened pole, and an equal diminution towards the dark pole. For the greater the Sun's altitude, the more directly its rays fall on any surface; and in surfaces of the same magnitude the quantity of light

light received by each is as the sine of the angle of obliquity with which the rays fall*.

D It remains to be shewn, that these relative situations of the axis and the terminator take place at different times of the year, with respect to the Earth; which being proved, the vicissitude of seasons must follow as a necessary consequence.

E In fig. 65. Let *c* represent the Sun, *ABDG* the Earth's orbit, nearly circular, but which being viewed obliquely, appears like a long ellipsis, of which let the part *BD* be supposed nearest the spectator. And let the four circles, distinguished by the months March, June, September, and December, represent the Earth in four several parts of its orbit, *NS* being its axis.

F Observation shews, that the axis of the Earth always preserves very nearly the same position with respect to the fixed stars; being inclined to the axis of its orbit in an angle of about $23\frac{1}{2}$ degrees. It will not therefore preserve the same relative position with respect to the terminator. For when the Earth is in the situation distinguished by the

* Let the line *AB* (fig. 64) represent a surface, on which the column of light *NOAB* falls perpendicularly. A surface *AC*, of the same magnitude, receiving the light obliquely under the angle *JCK*, will intercept only so much as would have fallen on the space *AB* and another surface *AD*, receiving the light still more obliquely under the angle *LDM*, will intercept only so much as would have fallen on the space *AF*. But the spaces or lines *AB* and *AF* are the sines of the angles of obliquity *JCK* and *LDM*; whence the proposition is evident.

month

month March, its axis will at that time be in the plane of the terminator, and consequently the days and nights will be equal all over the globe (183, *y*): but when by its annual motion it is carried towards *A*, the north pole *N*, the axis still preserving its position or continuing parallel to itself, will advance into the enlightened hemisphere, and in the month of June will be $23\frac{1}{2}$ degrees distant from the terminator, as in the scheme, the south pole being at the same distance in the dark hemisphere. Therefore in the month of June the northern parts will enjoy long days and summer, while the southern parts have short days and winter (184, *z*).

During the interval between the time of equal *a* days and nights in March, which is called the vernal equinox, and the time when the day is longest in June, which is called the summer solstice, the north pole will have described a quarter of a circle in the enlightened hemisphere with respect to the terminator, and consequently will be at its greatest distance from it. From that time it will, by describing the other quarter, approach the terminator, the days gradually shortening till the Earth arrives at the position denoted by the month September, when, the axis again coinciding with the plane of the terminator, the days and nights will be equal. This is called the autumnal equinox. During the next quarter the north pole will describe a quarter of a circle in the dark hemisphere, and the days will shorten till December, when the pole will be just as far within the dark

as in June it was in the enlightened hemisphere; which time is called the winter solstice: From the winter solstice to the vernal equinox, the days will lengthen as the pole approaches the terminator; and at the instant in which the axis again coincides with its plane, the natural year, consisting of 365 days, 5 hours, 48 minutes, and $45\frac{1}{2}$ seconds, is finished.

H It is easy to conceive, by applying the same explanation to the south pole instead of the north, that the inhabitants of the southern hemisphere have the same vicissitudes, though not at the same time; for it is winter in one hemisphere while it is summer in the other, &c. &c.

I As the pole N (fig. 62) advances in the enlightened hemisphere, the Sun will be in the zenith of a place G, as far distant from the equator, as the pole is from the terminator; therefore the greatest latitude at which the Sun can be vertical is $23\frac{1}{2}$ degrees. The parallels of latitude on the Earth of $23\frac{1}{2}$ degrees N. and S. as also the correspondent parallels of declination in the heavens, are called the Tropics, because the Sun when it arrives at them afterwards returns towards the equator. The Sun, when it arrives at the northern tropic, is just entering the sign Cancer, and when it arrives at the southern tropic is just entering the sign Capricorn; for which reason the northern tropic is called the Tropic of Cancer, and the southern tropic the Tropic of Capricorn.

C H A P. XIV.

OF THE FIXED STARS.

THOUGH in a former chapter of this section it was mentioned (104) that the relative situations of the fixed stars do not vary, yet that assertion is not to be understood in absolute strictness. In the course of ages several variations have been observed amongst them. Some of the larger stars have not the same precise situations that ancient observations attribute to them; and it is probable that the instances of this kind would have been much more numerous if accuracy of observation had not been confined to a very late period. New stars have likewise appeared from time to time, and several of those whose places and magnitudes are inserted in the old catalogues are not now to be found. Some of the fixed stars are likewise found to have a periodical increase and decrease of magnitude.

The bright stars Arcturus, Sirius, Aquila, and α Aldebaran, have been observed to change their places. The first is found to move towards the south, about $3\frac{1}{2}$ minutes of a degree in a century. Sirius has advanced about 2 minutes to the south in a like period. The changes of place in the two latter are yet smaller and less settled.

All the stars spoken of in the present chapter m are subject to no parallax (130, 2), according to the

the most accurate observations by which most of their places were settled; and some of them have been observed with instruments of such delicacy, that it is presumed their parallax would have been seen, if it had amounted to one second of measure.

- N** Without attending either to the celestial changes recorded by ancient authors, who for the most part were not astronomers, and passing the less obvious mutations in silence, we shall here note a few of the most remarkable new or changeable stars.
- O** On the 8th of November, 1572, Cornelius Gemma attentively considered that part of the heavens which is called Cassiope's Chair, and perceived nothing extraordinary. But the following night a new star appeared, forming a perfect rhombus with the three stars α , ϵ , γ , of that constellation. Its splendor exceeded that of Jupiter when greatest, and was such, that it was seen even in the day-time. Tycho Brahe, who saw it on the 11th, determined its longitude $6^{\circ} 54'$ of Taurus, with $53^{\circ} 45'$ N. latitude. It began to diminish in December, and became gradually less conspicuous, till it disappeared in March, 1574. This remarkable star had no apparent motion, and consequently no parallax, and its appearance was sparkling and clear, like that of the other fixed stars. It has not since been seen.
- P** September, 1604, O. S. the scholars of Kepler observed a star in the right leg of Serpentarius, which
was

was not there the night before. Its lustre seems to have been nearly equal to that of the new star in Cassiope; for it is described as exceeding Jupiter in brightness. It gradually decayed like that, and in nearly the same time disappeared, not being perceived after the beginning of January, 160 $\frac{1}{2}$. Its right ascension, as observed by Kepler, was constantly 256° 57', and its declination 2101 $\frac{1}{2}$ S.

The first star that was observed to have a periodical change of brightness was discovered by David Fabricius in the neck of the Whale, on the 3d of August, 1596, O. S. Its greatest brightness is equal to that of a star of the third magnitude; and it is scarcely ever so small but it may be seen with a six foot telescope. The period in which it passes through all its changes, is at a mean 334 days, but no part of the phenomenon is perfectly regular.

Three changeable stars have been observed in the constellation of the Swan. The first discovered is near the star γ in that constellation. Its greatest lustre is less than that of a star of the third magnitude, and it diminishes to that of the sixth magnitude. Its changes are far from being regular, and do not take place but after intervals of ten or more years.

The next and most remarkable of the changeable stars in the Swan is marked χ by Bayer. This is more regular in its returns than the preceding, through its magnitude is seldom greater than the sixth. Its period is settled at 405 $\frac{1}{6}$ days, and its greatest lustre in the year 1785 was about the 14th of July.

The

T The third was seen near the head of the Swan on the 20th of June, 1670, of about the third magnitude, and was so far diminished by the October following as to be scarcely visible. In the beginning of April, 1671, it was again seen rather brighter than before, and diminishing during that month, became once more at its greatest brilliancy at the beginning of May. By a comparison of these observations, its period seemed about ten months. It disappeared about the middle of August, and was again seen on the 29th of March, 1672; since which time it has not appeared.

U The star Algol, or Medusa's head, has been long since observed to appear of different magnitudes at different times; but the discovery of its period is due to John Goodricke, Esq; of York, who has observed it since the beginning of the year 1783. It periodically changes from the first to the fourth magnitude; and the time employed from one greatest diminution to the other, was Anno 1783, at a mean, 2 days, 20 hours, 49 minutes, 3 seconds. The change is thus. During four hours it gradually diminishes in lustre; during the succeeding four hours it recovers its first magnitude by a like gradual increase; and during the remaining part of the period, namely, 2 days, 12 hours, 42 minutes, 3 seconds, it invariably preserves its greatest lustre: after the expiration of which term the diminution again commences, &c.

V Many of the fixed stars, upon examination with the telescope, are found to consist of two. The
number

number observed before the time of Herschel* was but small; but that celebrated astronomer, who stands unrivalled for the excellence of his instruments, and his skilful industry in using them, has noted upwards of four hundred.

* Besides the phenomena already mentioned, there w are many nebulæ, or parts of the heavens which are brighter than the rest. The most obvious to common notice is that large irregular zone or band of light which crosses the ecliptic in Cancer and Capricorn, and is inclined to it in an angle of about sixty degrees. Other nebulæ are seldom so large as to be seen by the naked eye, to which they appear as small stars. If the telescope be applied to them, they seem to be luminous spots of various figures, in some instances with stars in them. The number of nebulæ ascertained before Herschel are about 103, and that observer had detected 466 more, previous to the month of April 1784. Many of the nebulæ are resolvable by the telescope into clusters of small stars; and it is found that telescopes of greater power resolve those nebulæ into stars, which appear as white clouds in instruments of less force. Hence there is good reason to conclude that they all consist of clusters or prodigious aggregates of stars.

Dr. Herschel has rendered it highly probable, x both from observation and well-grounded conjecture, that the starry heaven is replete with these nebulæ or

* Dr. Herschel's numerous and important discoveries are inserted in the late volumes of the Philosophical Transactions, as are also the accounts of Algol.

systems of stars of various figures; and that the milky-way is that particular nebula in which our Sun is placed. Nothing more is necessary in order to account for the appearance it exhibits, than to assume its figure as being much more extended towards the apparent zone of illumination than in other directions. And the observations on the various figures of the nebulae render this supposition perfectly allowable.

y The want of an annual parallax in the fixed stars evinces, that a luminous body, whose diameter is equal to that of the Earth's annual orbit, would not subtend a sensible angle if seen from the fixed stars. Much less therefore would the Sun, if viewed from such a distance. It may therefore be fairly concluded that the Sun, when seen from any fixed star, must have much the same appearance as a fixed star seen from hence: or, in other words, the fixed stars are suns.

z Reasoning then analogically, as far as the nature of the facts we possess will admit, it may be deduced; first, that the universe consists of nebulae, or distinct systems of stars; secondly, that each nebula is composed of a prodigious number of suns, or bodies that shine by their own native splendor; thirdly, that each individual sun is destined to give light to hundreds (124, R. 161, W. 164, E) of worlds that revolve about it, but which can no more be seen by us, on account of their great distance, than the solar planets can be seen from the fixed stars.

Yet,

Yet, as in this unexplored, and, perhaps, unexplorable abyss of space, it is no necessary condition that the planets should be of the same magnitudes as those belonging to our system, it is not improbable but that planetary bodies may be discovered among the double and triple stars. •

Our curiosity is much interested in the contemplation of the phenomena of new and changeable stars, but the causes that may be offered with plausibility to solve these appearances are not many.

If the light of the Sun and fixed stars be imagined to proceed from a combustion similar to that which is required to produce light in our experiments, it may happen that when all the inflammable matter is decomposed the ignition may cease. Or, if a mass of matter adapted for inflammation begin by any cause to burn, its ignition and emission of light will then commence. These considerations may explain the disappearance of some stars, and the appearance of others. And as there are no data to fix the time between the beginning and end of the appearance, the stars may last for any given time, according to circumstance..

The spots on the Sun (174, v) have afforded a conjecture respecting the cause of the periodical change of brightness in some stars. For, if a star be supposed to have a spot of considerable magnitude, and to turn on its axis, it will be much brighter when the spot is not on the visible disc than when it is. However, it must be confessed, that the phenomena do not in general agree with this supposition,

which cannot easily be reconciled to the permanent brightness or obscurity that prevails in the changeable flars for more than half the period.

- E If a star, by a swift rotation, be made to assume and preserve a flattened figure, and its axis have a rotation similar to that hereafter to be explained in the Earth, it will be much less bright when its edge is presented to the observer than when the visible disc is projected broader.
- I Or, lastly, if a large planet revolve about a star, it may occasion certain periodical eclipses of sufficient magnitude and duration to be perceived by us, on account of the quantity of light intercepted. Thus, for example if an opaque planet, whose diameter is not much less than that of Algol be supposed to revolve about that star at the distance of thirty-three diameters of Algol, in the given period of 2 days, 20 hours, 49 minutes, 3 seconds, in an orbit whose plane passes at present through or near the Earth, it will cause certain eclipses that will agree very well with the appearances observed.

B O O K I.

S E C T. IV.

Concerning the Physical Causes of the Celestial Motions.

C H A P. I.

OF THE GENERAL EFFECTS OF CENTRIPETAL FORCES ON BODIES IN MOTION.

WE are now to resume the consideration of bodies in motion, which are acted on by a centripetal force (94, 1) and to apply that doctrine to the phenomena explained in the preceding section.

Let $ABCD$, &c. (fig. 66) be a regular polygon, α inscribed in a circle. If a body be supposed to revolve in that polygon, it will be necessary that a force directed to the center s should be applied at the points B, C, D , &c. to deflect the body from its right lined course (21, P). The more numerous the sides of the polygon are, the nearer they will be to the circular curve, and the more frequent must be the successive actions of the centripetal force. And if the number of sides be infinitely great, the polygon will absolutely coincide with the circle, or become a circle,

and the actions of the centripetal force must be infinitely numerous, or the force will act without intermission. Consequently, whatever may be proved in general of a body moving in a regular polygon by an original uniform motion, combined with the motion produced by the successive actions of a force directed to the center of the polygon, will hold good with respect to the motion of a body in a circle, the centripetal force being supposed to act without intermission.

D In our reasoning concerning centripetal forces, it is here supposed that a given force acts on bodies according to their masses, like gravity (26, A) and consequently causes equal deflections in each from the right lined course.

E The intensity or quantity of any force is measured by the effect it produces in a given time (21, Q. 38, T). Suppose a body to be projected from M to A, fig. 66, a centripetal force represented by A R will cause it to describe the line A B instead of A Q in an equal space of time (23, T). If the velocity in M A had been greater, the action of the force in the line A R must have been greater in the same proportion to have caused the same deflection; that is, the centripetal force must at each point of deflection be as the velocity. But again, the greater the velocity the greater number of sides of the polygon will be described by the body in a given time, and the more frequent must be the actions of the centripetal force. For this reason therefore, the number of the actions must likewise be as the velocity. On both accounts, therefore,

therefore, the whole effect of the force in a given time, or its intensity, must be in the duplicate proportion, or as the square of the velocity, that is, simply as the velocity, because the actions themselves are greater or less in that proportion; and again, simply as the velocity, because the actions take place more or less frequently in the same proportion. And the same is true of the effect of an unceasing force that may cause the body to revolve in the circle in which the polygon is inscribed (198, c).

The polygon NOF being similar to the polygon ABC , will, with a given velocity, require the same action to cause the requisite deflections from the right lined course in a body revolving in it. But those actions must recur oftener in proportion as the side of the polygon NOF is less than that of the polygon ABC , because a proportionally greater number of the smaller sides will be passed over with the same velocity. The force in the smaller polygon must therefore be increased in the inverse proportion of its side to that of the greater, or, which is the same, in the inverse proportion of their radii. On the whole then, the centripetal forces, by which bodies are retained in circular orbits, (198, c) are in a ratio compounded of the direct ratio of the squares of the velocities, (199, F. G) and the inverse ratio of their semi-diameters.

The periodical times of bodies revolving in circles are greater, the greater the radii, and less, the

greater the velocities. That is to say, the periodical times are directly as the radii, and inversely as the velocities.

L If the centripetal forces be supposed to increase as the cubes or third powers of the radii decrease, the cubes of the radii will be inversely as the squares of the velocities, and directly as the radii, because the forces themselves are inversely in this compound ratio (199, 1). Whence the squares of the velocities will be directly as the radii, and inversely as the cubes of the radii; or more simply, M as the squares of the radii inversely. And the velocities themselves will consequently be inversely as the radii.

N. If the squares of the periodical times be directly
 as the cubes of the radii, the cubes of the radii
 will be (199, κ) directly as the squares of the
 radii; and inversely as the squares of the velocities.
 Therefore, the squares of the velocities will be
 directly as the squares of the radii, and inversely
 as the cubes of the radii; or, more simply, as the
 radii inversely. Now the centripetal forces (199, ι)
 are directly as the squares of the velocities, and in-
 versely as the radii. Therefore, if in this last com-
 pound ratio we substitute the inverse ratio of the
 radii, instead of the direct ratio of the squares of
 the velocities, to which it is equal, we shall have
 the forces in the inverse ratio of the squares
 of the radii. That is to say, if the squares of the
 periodical times of bodies revolving in circles be
 directly

directly as the cubes of the radii, the centripetal forces will be inversely as the squares of the radii.

The velocity of a body moving in a curve, and P acted on by a centripetal force, is inversely as the perpendicular let fall from the center, to the tangent drawn through that point of the curve at which the velocity is required. Let AB , fig. 67, be a curve in which a body moves, describing equal areas in equal times (95, M), about the point c . Then the velocity at the points D and F will be inversely as the perpendiculars cH , cI , let fall from c on the tangents DH , FI to the curve in the points D , F . For if the body move through the spaces DE , FG , in equal indefinitely small portions of time, those lines may be taken for portions of the tangents DH , FI , and the triangles DCE , FCG , will be equal (95, M). But the bases DE , FG , of equal triangles, are inversely as the perpendiculars cH , cI . And velocities being as the bases DE , FG , described in equal times, must also be in the same inverse ratio of those perpendiculars. Which was to be shewn.

If a body, acted on by a centripetal force, directed to c (fig. 68), be projected from u , in a direction at right angles to uc , but with a velocity too small to cause it to revolve in a circle uA , it will fall within the circle, by the greater prevalence of the centripetal force. As it approaches the center, its velocity must increase (96, R), and so must likewise its tendency to recede from the center.

center. If the centripetal force increases in the same or in a higher ratio than that tendency, the body will still continue to approach, and at length fall into the center. But if the centripetal force increases in a less ratio, the increasing velocity will cause the body to move in a course less and less inclined to the radius, till at length it becomes at right angles to it, and recedes again from the center, because by the supposition, the velocity is too great for the body to move there in a circular orbit. In the recess from the center the velocity must decrease (96, R), and a similar curve be described by the body, till its course becomes again at right angles to the radius, and it is again caused to approach the center. And this alternation will continue for ever.

- R The assertion respecting the similarity or rather congruity of the curves between the apsides may be easily evinced from fig. 40. For if a body be supposed to move from H to c, and to be reflected back from c, in the direction and with the velocity cD, it is unnecessary to shew that it would again describe the same polygon cH. And the same holds good of curves (95, M). Now a body that arrives at its apsis must move with a velocity and direction which, with respect to the center, is equivalent to its being reflected back in the contrary direction, because in either case it will begin to move with a given velocity in the tangent of the same circle.

It

It would carry us too far into the consideration of the nature of those curves that may be described by bodies acted on by centripetal forces, if we were to enquire minutely into the consequences that would follow from the supposition of various laws of its increase or diminution, according to the distance. Our purpose will be sufficiently answered by attending to the velocities of revolving bodies in their apsides. Let a body (fig. 68) be projected from u towards A , in a direction at right angles to cu , a line drawn from the point c , to which let the centripetal force be supposed to be directed. Suppose the velocity of projection to be less than would be required to carry the body in a circle at u , and the body will accede towards the center, by passing through a curve $u D F$. If the law of the centripetal force be such (201, Q.) as to suffer the body to recede again, after coming within a certain distance of the center, there will be some point L , at which the body, previous to its going off, will neither approach nor recede from the center, but move in a direction at right angles to the radius. This point is the lower apsis, and its velocity will then be inversely as the perpendicular $c L$ (201, P).

Let us suppose the centripetal force to be inversely as the cube of the distance from c . Then (200, M) the velocities necessary to carry a body in a circle at u or L will be inversely as the distances uc , Lc . The actual velocities at u and L are in the same ratio, and the velocity at u is known to be

- be too small to carry it in a circle there. Consequently, the actual velocity at L must be likewise too small in the same ratio, and the body will continue to approach the center, after having passed the apsis. It will not, therefore, describe a curve congruous with the curve described in its passage between the two apsides. But this last consequence being contrary to what has been already proved (202, R) must be false, and so must likewise be the original supposition from which it was deduced.
- v Consequently it is not true, that a body projected with a velocity too small to keep it in a circle, and acted on by a centripetal force inversely as the cubes of the distances, can ever arrive at the lower w apsis. It must therefore continually approach the center, and at length fall into it.
- x If the body be supposed to be originally projected from L , the lower apsis, with a velocity too great for the centripetal force, according to the same law, to retain it in a circle, it may be shewn by similar argumentation that it would never arrive at the higher apsis, but would continually recede from the center.
- y Thus it appears, that the inverted ratio of the cubes of the distances is the law of centripetal force that limits the revolutions of bodies in curves that admit of alternate approach and recess from the center. For if, according to this law, a body, after once beginning to approach the center or to recede from it, cannot but continue that approach or recess, it must be much more strongly urged in the same manner

manner by a centripetal force that follows the inverted ratio of some higher power of the distance. And again, if the law of the force follows some inverted ratio less than that of the cube of the distance, the velocities required to retain bodies in circular orbits will be less than after the inverted ratio of the distance as that law would require (200, L, M). Whence it follows, that since the velocities in descending from the upper apsis increase faster than the distances decrease, the perpendicular CH , fig. 67, being less than the distance CD (201, P), the motion of the body will be directed less and less towards the center, till it becomes at right angles to the radius, the body being then in the lower apsis. After which it must ascend through a curve similar and equal to that it before described in passing between the apsides (202, R).

Hence it is seen, that when the centripetal force z increases in approaching the center in a less ratio than the inverse ratio of the cubes of the distances, the law of its increase may be found from the quantity of angular motion employed in passing from one apsis to the other. For the distance between A the upper and lower apsis will be greater the nearer the law of the centripetal force approaches to that ratio, because the body must run through a greater space before the tendency to recede from the center, arising from the velocity and direction, can be equal to the centripetal force.

If a body revolves in an elliptical orbit, describing equal areas in equal times about one of the foci,

foci, the apsidal will be at the two extremities of the transverse diameter, or 180° of angular motion apart, the centripetal force will be directed to that focus (95, N), and its intensity will be inversely as the square of the distance*.

- C If a body revolves between two apsidal, and the centripetal force be inversely as some power of the distance, greater than the square and less than the cube, the distance between the apsidal will be more than 180° . But if the centripetal force be inversely as some power of the distance less than the square, the distance between the apsidal will be less than 180° (205, A). In these cases the orbit may be considered as an ellipsis whose transverse diameter, or line of the apsidal, is not stationary, but revolves on the focus to which the force
- D is directed. The apsidal may therefore be said to revolve in consequentia, or with the moving body, when the force in approaching the center is greater than after the inverse ratio of the square of the distance; or to revolve in antecedentia when the
- E force is less than after that ratio. And the quiescence of the apsidal will be a proof, that the centripetal force is accurately in the inverse ratio of the square of the distance.
- F The periodical time of a revolution about the focus in a quiescent ellipsis is equal to that which would be employed in describing a circle whose radius is half the transverse diameter of the ellipsis †.

* Principia, I. §§. 3. 9.

† Principia, I. 15.

Therefore,

Therefore, if the squares of the periodical times of bodies revolving in ellipses be directly as the cubes of the mean distances (200, o) the centripetal forces will be inversely as the squares of the distances.

C H A P. II.

THE UNIVERSALITY OF GRAVITATION DEDUCED FROM ITS EFFECTS.

THE planetary bodies being in motion would (21, P) continue to move for ever in right lines, unless compelled to change their state by forces impressed. But they move in curve lines (121, K), and consequently must be acted on by forces that continually deflect their courses out of the right-lined direction.

Every primary planet moves with such a velocity and direction, that a line joining the centers of the planet and the Sun describes equal areas in equal times (121, K). Whence it follows, that the centripetal forces which retain these planets in their orbits are (95, N) directed to the Sun's center.

The periodical times of the primary planets are such, that their squares are directly in proportion to the cubes of their mean distances from the Sun. Their orbits are (121, K) elliptical, and their apsidal quiescent. From these phenomena it is proved K (207, G. 205, B. 206, E), that the centripetal forces are inversely as the squares of the distances from the Sun.

Every secondary planet moves with such a velocity and direction, that a line joining the center of the secondary with that and its primary, describes equal areas in equal times. The centripetal forces retaining these bodies in their orbits consequently are (95, N) directed to the centers of their respective primaries.

The periodical times of Saturn's moons are such that their squares are directly in proportion to the cubes of their distances. And, therefore, the centripetal forces are inversely as the squares of the distances (200, O).

The same phenomenon in Jupiter's moons shew, that their centripetal forces follow the same law.

The orbits of Saturn's and Jupiter's moons are here taken to be circular. For observation has not yet established the eccentricity of any of these orbits, except that of Jupiter's fourth satellite.

The Moon is carried about the Earth with such a velocity and direction, that a line joining its center and that of the Earth, describes equal areas in equal times (143, P). It is therefore retained in its orbit by a force directed to the Earth's center.

The Moon's orbit is elliptical, and its apsidal quiescent. Its centripetal force is therefore inversely as the square of its distance (205, B. 206, R).

Every comet moves with such a velocity and direction, that a line joining the centers of the comet and Sun describes equal areas in equal times

times (160, v). The centripetal forces retaining ^R the comets in their orbits is (95, N) therefore directed to the Sun.

All the comets are observed to describe either ellipses, or parabolas, which in all probability are the lower portions of ellipses (160, u, v). Those whose return has been observed have their apses quiescent. Whence it follows * (205, B. 206, E), ^S that the centripetal forces are inversely as the squares of the distances from the Sun.

It is not to be understood that the planetary ^T phenomena are in absolute strictness as given in this place. But the irregularities are very small, and it will hereafter be seen, that they are of such a nature as to give additional force to the deductions here made.

The force that retains the Moon in its orbit is ^U the same with that which causes bodies near the Earth's surface to be heavy, and is called Gravity.

To prove this important truth, let us take 57' ^V for the Moon's horizontal parallax at its mean distance, and that distance will, by plane trigonometry, be found to be 60.314 semidiameters of the Earth. The periodical time of the Moon is 27 d. 7 h. 43 m. (142, A), or 39343 minutes, which is the same period as would obtain if its orbit were a circle (206, F) whose radius is equal to the mean distance, and the centripetal force remained unaltered. To come at the effects of this force more

* For the parabola, see Principia, I. 13.

readily, it will be convenient to attend to this circular revolution. In one minute of time the Moon in this orbit would pass through the $\frac{1}{35311}$ part, or an arc of 32.941 seconds of measure. The mean length of a degree on the Earth's meridian is 342516 Paris feet, which number multiplied by 60.314, will give 20658510 Paris feet for the length of a degree in the supposed circular orbit. Whence the arc of 32.941 seconds passed through in a minute may be found in feet, as also its versed sine. The versed sine being the space through which the Moon must fall beneath the tangent in the time of one minute, will be the effect of, and will measure the centripetal force (35, E). This space or versed sine is 15.0944 Paris feet. Now, because the Moon's centripetal force is inversely as the square of the distance from the Earth's center (208, Q), we may find what its effect would be at the Earth's surface by saying, As the square of 60.314, the Moon's distance from the Earth's center; Is to the square of 1, the distance of the surface of the Earth from its center: So is the measure (35, E) of the centripetal force at the Moon 15.0944 feet; To the measure or effect of the same force at the Earth's surface, or 54910 feet.

w To find whether this space agrees with the fall of bodies at the Earth's surface by gravity, we must reduce the time to one second; because we have no means of directly measuring the actual fall of bodies during so long a time as one minute. Now, though

though the centripetal force of the Moon increases in approaching the Earth's center, yet these measures of that force are strictly accurate, because considered as they obtain in circular orbits where the distance of the body is not diminished by its fall: but the difference would in the present case be absolutely insensible in so short times as a second or a minute, even if we supposed the falling body to be moved in a right line directed to the center. The spaces described by falling will consequently be described by an uniformly accelerated motion, and will be (29, 6) as the squares of the times. Therefore as the square of 60 seconds; Is to the square of 1 second: So is 54910 feet; To 15.2528 feet that bodies would fall through at the Earth's surface in a second by the action of the centripetal force that retains the Moon in its orbit.

But bodies fall through 15.084 Paris feet in a x second, by the action of gravity. The fall of bodies near the Earth's surface and of the Moon are effects of the same kind, and therefore, by the second rule of philosophizing, are produced by the same cause. That is to say, the Moon is retained in its orbit by gravity.

Moreover, since it is established from the Moon's y revolution in its orbit that a centripetal force exists and acts in the direction towards the center of the Earth, and no good reason can be given against its action that would take place, according to its law, on bodies any where situated, it must follow, that bodies fall near the Earth's surface, ei-

ther by this force alone, or by this force in conjunction with some other. But this latter consequence cannot be admitted, because bodies would then fall with greater velocity than the Moon at the same distance; whereas it has been just shewn, that their velocities are somewhat less.

- z And even this difference between the velocities of the Moon and of smaller bodies affords an additional proof, that the same force of gravity is concerned in both. For the force would have proved accurately the same, as far as observation can measure its effects, if proper allowance, according to the laws of gravity known from its effects on heavy bodies, had been made in the computation for the mass of the Moon, and the Sun's action on the Earth and Moon.
- A The revolutions of the satellites of Jupiter and Saturn, and also those of the primary planets and of the comets, are phenomena of the same kind as the Moon's revolution about the Earth, and are therefore (6, 11) to be attributed to the same causes, namely, to an original or projectile motion compounded with a motion produced by gravity.
- B Action and re-action (22, 2) being equal, it follows also, because the secondary planets gravitate towards their primaries, and the primary planets together with their satellites, as likewise the comets, gravitate towards the Sun, that the primary planets must likewise gravitate towards their secondaries, and the Sun towards the whole
- c system. That is to say, gravitation is universal, or a property of all bodies whatsoever.

The

The mutual action of two bodies (22, R. 95, O) ^D that gravitate to each other is the cause that if they fall, both will approach the common center of gravity with equal quantities of motion, or if they have at the same time a projectile velocity, both will absolutely revolve about that center. If one of the two bodies exceed the other indefinitely in mass, its velocity (19, L) of approach will be indefinitely less than that of the other in the former case, or the radius of its orbit will be indefinitely less than that of the other in the latter case. Whence the whole relative velocity of ap- ^E proach may be taken for the absolute velocity produced in the less body; or the orbit described by the less body about the greater may be taken for that which it describes about the common center of gravity.

The fall of bodies near the Earth's surface may ^F be regarded as their absolute motion. For the magnitude of the Earth is so great with respect to the bodies with which art can make experiments, that its velocity is incomparably smaller than the differences which the imperfections of the senses must cause in all observations. We may ^G likewise in this place consider the motions of the planets and comets about the Sun as absolute, though the magnitude of that luminary is not so excessive as to render those of the planets inconsiderable. The same is to be understood of the ^H satellites of Jupiter and Saturn.

All bodies fall near the Earth's surface with ^I equal velocities (26, A, B). The planets and comets being accelerated towards the Sun by powers which

are (207, K. 209, s) inverfely as the fquares of the diftances, would confequently be equally accelerated at equal diftances. The fame is true of the fatellites with refpect to the Sun, becaufe they revolve together with their primaries in the annual orbits, and alfo with refpect to their primaries (208, M, N).

K From thefe equal accelerations it follows, that the force of gravity which urges minute bodies towards any other larger body, is in proportion to the mafs of the body urged.

L This law of gravity being eftablifhed from obfervations in cafes where the velocity of the fmall body can be taken without fenfible error, we may again refume the confideration of the re-action of the fmall. Let us call the larger L, and the fmall s. Then, becaufe s is urged towards L, by a force which is as the magnitude of s, L will be urged (22, R) towards s, with the fame force. That is, if s becomes larger, it will attract L the more ftrongly in proportion. Now, s may be imagined to become larger, fo as even to exceed L in any ratio whatever, and the increafe of attraction will ftill obtain. Therefore, a given body not only attracts another, in proportion to the mafs of this laft, but alfo in proportion to its own mafs. That is to fay, the force of gravitation exerted between two bodies is in the compound ratio of their maffes.

O The abfolute force of gravity being in the compound ratio of the maffes of the two bodies, or as the maffes multiplied into each other, will be
meafured

measured by the quantity of motion produced in either body (21, Q. 19, L) in a given time, or by the mass of the body multiplied into its velocity; that is, the numeral product of the masses will be equal to the numeral product of the mass of one of the bodies into its velocity. Now, if each of these equal products be divided by the mass of the body, whose velocity is considered, the two remainders will be equal, namely, the mass of the one body will be equal to the velocity of the other. Or, more P clearly (because we have used number, in order to avoid the comparison of ratios, which is less generally understood) the mass of one body will be as the velocity or acceleration of the other. Whence Q it follows, that if the velocities produced in bodies by gravitation be known, the proportional masses of the bodies towards which they are urged will be also known.

From this consequence the mass of any large pla- R net may be known from the velocity of descent it produces in bodies indefinitely smaller than itself. For the relative velocities of such small bodies (213, E) with respect to the larger, may be taken for their absolute velocities. Now, the velocities of the planets : towards the Sun, of the satellites of Jupiter and Saturn towards their respective primaries (213, G), and of projectiles near the Earth's surface, being reduced to equal distances of the attracting bodies, as was done in comparing the Moon's gravity with that of terrestrial bodies (209, v, w, x) or otherwise, are, and consequently likewise their masses are, (nearly)

as the numbers 1, $\frac{1}{1687}$, $\frac{1}{3927}$, and $\frac{1}{168712}$. Hence also the densities may be found, because their bulks
 T are known from observation. Thus, the densities, or specific gravities, are in the four bodies just mentioned, 100, $94\frac{1}{2}$, 36, and 400. These numbers may be familiarized to the imagination, by observing, that if the mean density of the Earth be supposed to answer to that of common green glass, the Sun's density would be equal to that of dry pear-tree, Jupiter's to cedar, and Saturn's to cork.

U Our knowledge of the remote parts of the planetary system is too imperfect to admit of many re-
 V marks on the facts we can observe and deduce. It is, however, worthy of notice, that the immensely large planets, Jupiter, Saturn, and the Georgium Sidus, would have occasioned great irregularities by their attractions on the other bodies of the system; if, instead of being placed at the great distances they are from the common center of gravity, and from each other, they had occupied the places of the small planets, Mars, the Earth, Venus or Mercury. And if the celestial spaces be not absolutely
 W vacuous, but possessed by some very rare matter, (175, A) the resistance such matter must afford in the course of ages to the motions of the planets, will be brought nearly to equality in its effects, if the planets which move swifter are at the same time more dense. Lastly, because it is observed in the constitution of terrestrial bodies, that the denser require, in many instances, a higher degree of heat to produce given changes in them, it has been con-
 jectured,

jectured, not without some degree of probability, that the planets nearer the Sun are for this purpose formed of denser materials adapted to their situation.

From the universality of gravitation (212, c) x it is deduced, that the fixed stars are either falling towards the common center of gravity of the universe, or are made to describe immense orbits which ultimately respect that center. The life of man, assisted by every traditional record for thousands of years, seems too short to ascertain the result of this sublime enquiry. It will be a grand acquisition, if the repeated observations of several centuries to come should determine the proper motions of the vast number of suns that compose the nebula, (194, z) of which our whole planetary system, with all its comets, forms so inconsiderable a part.

C H A P. III.

OF THE IRREGULARITIES ARISING FROM THE
MUTUAL GRAVITATION OF THE PLANETS.

- Y** If the Sun were at rest, and the planets did not mutually gravitate towards each other, they would describe ellipses, having the Sun in the common focus. But since they mutually act on the Sun, and on each other (212, c) it must follow that the Sun is perpetually moved about the center of gravity of all the planets, which center is the common focus of their orbits. This center, by reason of the Sun's very great bulk, can, in no situation, exceed the distance of its semidiameter from its surface.
- Z** Some small irregularities arise from these mutual actions, but much less than would ensue if the Sun were at rest, or not subject to the re-action of the other planets. The irregularities in the motions of the primary planets are scarcely considerable enough to come under observation in the course of many revolutions: those of the Moon, on account of its nearness to us, and from other causes, have ever been sufficiently great, to embarrass the astronomical world. It will therefore be sufficient to explain the latter, and apply the explanation to the former, which are effects of the same kind.
- H** If the actions of the Sun upon the Earth and Moon were equal upon each, according to their masses, and tended to produce motions in parallel directions,

directions, their relative motions would be the same as if no such forces acted upon them (79, w). But these forces vary, both in quantity and direction, according to the various relative situations of the Earth and Moon.

Let the point s (fig. 66) represent the Sun, E the Earth, and $A D B C$ the orbit of the Moon. Then, if the Moon be at the quadrature A , the distances Es and As of the Earth and Moon from the Sun, being equal, their gravities towards s will also be equal, and may be represented by those lines Es and As . Draw the line AL parallel and equal to Es , and join Ls , which will be parallel to AE . The force As may be resolved (23, T) into the two forces AL and AE ; of which AL , by reason of its parallelism and equality to Es (79, w) will not disturb their relative motions or situation: but the force AE , conspiring with that of gravity, will cause the Moon to fall farther below the tangent of its orbit than it would have done if no such disturbing force had existed. Therefore, at or near the quadratures, the Moon's gravity towards the earth is increased more than according to the regular course, and its orbit is rendered more curve.

When the Moon is at the conjunction c , the distances Es and cs not being equal, the Moon's gravitation towards the Sun exceeds that of the Earth in the same proportion as the square of Es exceeds the square of cs . And because the excess acts contrary to the direction of the Moon's gravity towards the Earth; it diminishes the effect thereof, and causes

causes the Moon to fall less below the tangent of its orbit than it would if no such disturbing force existed. A like, and very nearly equal, effect follows, when the Moon is at the opposition ν , by the Earth's gravitation towards the Sun being greater than that of the Moon: whence their mutual gravity is diminished as in the former case. Therefore, at or near the conjunction or opposition, the Moon's gravity is diminished, and its orbit is rendered less curve.

F It is found, that the force added to the Moon's gravity at the quadratures, is to the gravity with which it would revolve about the earth in a circle at its present mean distance, if the Sun had no effect on its motion, as 1 to 190; and that the force subtracted from its gravity at the conjunction or opposition is about double this quantity. The influence of the Sun, then, on the whole, increases the Moon's distance from the Earth, and augments its periodical time; and since this influence is most considerable when the Earth is nearest the Sun, or in its perihelium, its periodical time must then be the greatest, as appears likewise from observation (144, K).

H To shew the effect of the Sun in disturbing the Moon's motion at any situation between the conjunction and one of the quadratures, suppose at m (fig. 66) let es represent the Earth's gravity towards the Sun; draw the line ms , which continue towards g ; from m set off mg , so that mg may be to es as the square of the Earth's distance es is to the

the

the square of the Moon's distance MS ; and MG will represent the Moon's gravity towards the Sun. From M draw MF parallel, and equal to ES ; join FG , and draw MH parallel, and equal to FG . The force MG may be resolved into MF and MH ; of which MF , by reason of its parallelism and equality to ES , will not disturb the relative motions or situations of the Moon and Earth; MH then is the disturbing force. Draw the tangent MK to the Moon's orbit, and continue the radius EM towards I ; draw HI parallel to KM , and intersecting MI in I , and complete the parallelogram by drawing HK parallel to IM , and intersecting MK in K . The force MH may be resolved into MI and MK ; of which MI affects the gravity, and MK the velocity of the Moon. When the force MH coincides with the tangent, that is, when the Moon is $35^{\circ} 16'$ distant from the quadrature, the force MI , which affects the gravity, vanishes; and when the force MH coincides with the radius, that is, when the Moon is either in the conjunction or quadrature, the force MK vanishes. Between the quadrature and the distance of $35^{\circ} 16'$ from it, the line or force MH falls within the tangent, and consequently the force MI is directed towards E , and the Moon's gravity is increased: but, at any greater distance from the quadrature, the line MH falls without the tangent, and the force MI is directed from E , the Moon's gravity being diminished. It is evident that the force MK is always directed to some point in the line which passes through the centers of the Sun and Earth; therefore

therefore it will accelerate the Moon's motion, while it is approaching towards that line, or the conjunction, and similarly retard it as it recedes from it, or approaches towards the quadrature, by conspiring with the motion in one case, and subducting from it in the other.

K As the Moon's gravity towards the Sun at the conjunction is diminished by a quantity which is as the difference of the squares of their distances; and as this difference, on account of the very great distance of the Sun, is nearly the same when the Moon is at the opposition, the mutual tendency to separate, or diminution of gravity, will be very nearly the same. Whence it easily follows, that all the irregularities which have been explained as happening between the quadratures and conjunction must in like circumstances take place between the quadratures and the opposition.

L If the Moon revolved about the Earth in a circular orbit, the Sun's disturbing influence being supposed not to act, then this influence being supposed to act would convert the orbit into an ellipsis. For the increase of gravity renders it more curve at the quadratures, by causing the Moon to fall further below the tangent; and the diminution of gravity, as well as the increasing velocity, renders the orbit less curve at the conjunction and opposition, by causing the Moon to fall less below the tangent in a given time. Therefore an ellipsis would be described, whose less or more convex parts would be at the quadratures, and whose longest diameter would

would pass through them. Consequently the Moon would be farthest from the Earth at the quadratures, and nearest at the conjunction and opposition. Neither is it strange that the Moon should approach or come nearer to the Earth at the time when its gravity is the least, since that approach is not the immediate consequence of the decrease of gravity, but of the curvity of its orbit near the quadratures; and in like manner, its recess from the Earth at the quadratures does not arise immediately from its diminished gravity, but from the velocity and direction acquired at the conjunction or opposition.

But as the Moon's orbit is, independent of the Sun's action, an ellipsis, these effects take place only as far as circumstances permit.

The Moon's gravity towards the Earth being thus subject to a continual change in its ratio, its orbit is of no constant form. The law of its gravity being nearly in the inverse proportion of the squares of the distances, its orbit is nearly a quiescent ellipsis (206); but the deviations from this law occasions its apsides to move direct or retrograde, according as those deviations are in defect or excess (206, c). Astronomers, to reduce the motion of the apsides to computation, suppose the revolving body to move in an ellipsis, whose transverse diameter or line of the apsides revolves at the same time about the focus of the orbit. When the Moon is in the conjunction or opposition, the Sun subducts from its gravity (220, e), and that the more the greater its distance is from the Earth, so that

that its gravity follows a greater proportion than the inverted ratio of the square of the distance, and consequently the apsidal of its orbit must then move in consequentia, or direct (206, D). In the quadratures the Sun adds to the Moon's gravity (219, O), and that the more the greater its distance from the Earth, so that its gravity follows a less proportion than the inverted ratio of the square of her distance, and consequently the apsidal of its orbit must then move in antecedentia, or retrograde (206, E). But because the action of the Sun subducts more from the Moon's gravity in the conjunction and opposition than it adds to it in the quadratures (220, F) the direct motion exceeds the retrograde, and at the end of each revolution the apsidal are found to be advanced according to the order of the signs.

P If the plane of the Moon's orbit coincided with that of the ecliptic, these would be the only irregularities arising from the Sun's action; but because it is inclined to the plane of the ecliptic in an angle of about five degrees, the whole disturbing force does not act upon the Moon's motion in its orbit, a small part of the force being employed to draw it out of the plane of the orbit into that of the ecliptic.

Q Of the forces mk and mi , fig. 66, which disturb the Moon's motion, mi being always in the direction of the radius, can have no effect in drawing it out of the plane of its orbit. And if the force mk really coincided with the tangent, as we, neglecting

lecting the small deviation arising from the obliquity of the Moon's orbit have hitherto supposed, it is evident that its only effect would be that of accelerating or retarding the Moon's motion, without affecting the plane of its orbit. But because that force is always directed to some point in the line which passes through the centers of the Sun and Earth (221, 1) it is evident that it can coincide with the tangent only when that line is in the plane of the Moon's orbit; that is to say, when the nodes are in the conjunction and opposition. At all other times the force mk must decline to the northward or southward of the tangent, and compounding itself with the Moon's motion, will not only accelerate or retard it, according to the circumstances before explained, but will likewise alter its direction, deflecting it towards that side of the orbit on which the point, the force mk , tends to, is situated. This deflection causes the Moon to arrive at the ecliptic either sooner or later than it would otherwise have done; or, in other words, it occasions the intersection of its orbit with the ecliptic to happen in a point of the ecliptic, either nearer to, or further from, the Moon, than that in which it would have happened if such deflection had not taken place.

To illustrate this, let the elliptical projection $RCOQN$ (fig. 67) represent a circle in the plane of the ecliptic, $MOFN$ the Moon's orbit, intersecting the ecliptic in the nodes N and O . Suppose the Moon to be in the northern part of its orbit at M , and moving towards the node O ; the disturbing force

mk , which tends towards a point in the line se to the southward of the tangent mt , will be compounded with the tangential force, and will cause the Moon to describe the arc mm , to which mk is tangent, instead of the arc mo ; whence the node ϕ is said to be moved to m . In this manner the motion of the nodes may be explained for any other situation.

- s This motion evidently depends on a twofold circumstance, namely, the quantity and direction of the force mk . If the force mk be increased, its direction remaining the same, it will deflect the curve of the Moon's path from its orbit in a greater degree; and on the other hand, if its direction be altered, so as to approach nearer to a right angle with the tangent, it will cause a greater deflection,
- t though its quantity remain the same. When the Moon is in the quadratures, the force mk vanishes, (220, n) consequently the nodes are then stationary. When the Moon is at the octant, or forty-five degrees from the quadrature, the force mk is greatest of all, and therefore the motion of the nodes is then most considerable, as far as it
- u depends on the quantity of mk . But the direction of this force in like circumstances depends on the situation of the line of the nodes. If the line of the nodes coincides with the line passing through the centers of the Sun and Earth, the force mk coincides with the tangent of the Moon's orbit, and the nodes are stationary. And the farther the node is removed from that line, the farther is that
- line

line removed from the plane of the Moon's orbit; till the line of the nodes is in the quadratures, at which time the line passing through the centers of the Sun and Earth, makes an angle with the plane of the Moon's orbit equal to its whole inclination, or five degrees: consequently the angle formed between $m\kappa$ and the tangent in like circumstances is then greatest, $m\kappa$ being directed to a point in a line which is further from the plane of the Moon's orbit than at any other time, and of course the motion of the nodes is then most considerable.

To determine the quantity and direction of the v motion of the nodes, suppose the Moon in the quarter preceding the conjunction, and the node towards which it is moving to be between it and the conjunction: in this case its motion is directed to a point in the ecliptic, which is less distant than the point towards which the force $m\kappa$ is directed: the force $m\kappa$ then, compounding with the Moon's motion, causes it to be directed to a point more distant than it would otherwise have been; that is to say, the node, towards which the Moon moves, is moved towards the conjunction. When the Moon has passed the node, its course is directed to the other node, which is a point in the ecliptic more distant than the point to which $m\kappa$ is directed, and therefore $m\kappa$, compounding with its motion, causes it to be directed to a point less distant than it would otherwise have been; so that in this case likewise, the ensuing node is moved towards the conjunction. After the Moon has passed

the conjunction, the force mk still continues to deflect its course towards the ecliptic, and consequently the motion of the node is the same way till its arrival at the quadrature. Suppose again, the Moon to be at the conjunction, and the node towards which it is moving to be between it and the quadrature. In this case the force mk compounding with the Moon's motion, causes it to move towards a point in the ecliptic less distant than it would otherwise have done, so that the ensuing node is brought towards the conjunction. When the Moon has passed the node, the force mk still continuing to deflect its course towards the same side of its orbit, produces a contrary effect, namely, as it before occasioned it to converge to the ecliptic, so it now causes it to diverge from it, and its motion in consequence tends continually to a point in the ecliptic more distant than it would otherwise have done: the ensuing node in this instance being also brought towards the conjunction.

w As the disturbing forces are very nearly the same in the half of the Moon's orbit, (222, κ) which is farthest from the Sun, this last paragraph is true, when it moves in that part of its orbit, if the word opposition be every where inserted instead of the word conjunction.

x Whence it is easy to deduce this general rule, that when the Moon is in the part of its orbit nearest the Sun, the node towards which it is moving is made to move towards the conjunction: and when it is in the part of its orbit farthest from the Sun, the
node

node towards which it is moving is made to move towards the opposition.

Suppose the Moon at Q , (fig. 68) or the quadrature preceding the conjunction, then the ensuing node, if at 90° distance, or at the conjunction c , will be stationary (226, v), but if it be at a greater or less distance, it will be brought towards c (228, x). Thus, if the nodes be in the position MN , the ensuing node M , being at a less distance from Q than 90° , will move towards c , or direct, while the Moon moves through the arc QM ; after which N becomes the ensuing node, and likewise moves towards the conjunction c , or retrograde during the Moon's motion through the arc MR . And because the arc MR exceeds QM , the retrograde motion exceeds the direct. Again, if the nodes be in the position nm , the ensuing node n being at a greater distance from Q than 90° , will move towards c , or retrograde, during the Moon's motion through the arc Qn ; after which the node m becomes the ensuing node, and likewise moves towards the conjunction c , or direct, during the Moon's motion through the arc nR . And because the arc Qn exceeds nR , the retrograde motion here also exceeds the direct. If the nodes be in the quadratures QR , the ensuing node R removes towards c , or retrograde, during the Moon's motion through the arc QR , or almost the whole semi-orbit. The same may be shewn in the other half of the orbit ROQ with respect to the opposition o ; and therefore, in every revolution of the Moon, the retrograde motion of the nodes ex-

- Z** ceeds the direct; and, on the whole, the nodes are carried round, contrary of the order of the signs.
- A** The line of the conjunction is by the Earth's annual motion brought into every possible situation with respect to the nodes in the course of a year, independant of their own proper motion; which last occasions the change of situation to be performed in about nineteen days less.
- B** The inclination of the Moon's orbit being the angle which its course makes with the plane of the ecliptic, it is evident from what has been said, that this angle is almost continually changing. Suppose the line of the nodes, by its retrograde motion, to leave the conjunction *c*, fig. 69, and become in the second and fourth quarters as in the position *mn*, and the Moon to move from the node *m* to the node *n*; then, because the ensuing node *n* moves (228, x) towards the conjunction *c*, while the Moon is in the nearer half of its orbit, the Moon's course must be continually more and more inflected towards the ecliptic, till its arrival at *R*. This inflection in the first 90°, or *MA* from *M*, prevents its diverging so much from the ecliptic as it would otherwise have done; that is to say, it diminishes the angle of the Moon's inclination. From *A* to *R* its course begins to converge towards the ecliptic, and this convergence is increased by the inflection which in the preceding 90° prevented its divergence: in the arc *AR* then the inclination is increased. During the Moon's motion from *R* to *N*, the node is moved towards the op-
position

position *o*, and consequently the angle of its course to *n* is rendered less than it would have been if the node has not moved; or, in other words, the inclination is diminished. And because the arc *MA* added to the arc *RN* is greater than the arc *AR*, the inclination at the subsequent node is less than at the precedent node; and the same may be shewn in the other half revolution *NQM*. Therefore, *c* while the nodes are moving from the conjunction and opposition to the quadratures, the inclination of the Moon's orbit, on the whole, diminishes in every revolution till they arrive in the quadratures, at which time it is least of all. When the line of *D* the nodes has passed the quadratures, and is in the first and third quarters, as in the position *mn*, it is easily shewn by the same kind of argument, that the inclination is increased while the Moon passes from *m* to *q*, then diminishes for the remainder of the first 90° or *qa*, and is afterwards increased for the other 90° or *an*: and the same may be proved for the other half revolution *nRM*. Consequently, *E* while the nodes are moving from the quadratures to the conjunction and opposition, the inclination is increased by the same degrees as it before was diminished, till they arrive at the conjunction and opposition, at which time it returns to its first quantity, being then greatest of all.

The line of the nodes in the course of one entire revolution, with respect to the Sun, is twice *F* in the quadratures and twice in the conjunction and opposition. Therefore, the inclination of the

Moon's orbit to the ecliptic is diminished and increased by turns, twice in every revolution of the nodes.

- G All the irregularities of the Moon's motion are a little greater when in the half of its orbit nearest the Sun, than when it is in the other half; the chief reason of which is, that the difference between the squares of the Moon's and Earth's distances from the Sun is greater, in proportion to the squares themselves, in the former than in the latter case at equal elongations from the quadrature, and consequently the disturbing forces must be more considerable.
- H Although the Moon in reality revolves about the common center of gravity between itself and the Earth, and not about the Earth itself, and consequently their motions and irregularities are similar, and not confined to the Moon alone; yet it may be easily conceived, that the conclusions are not affected in any degree that may be here regarded, when, for the sake of conciseness, we suppose one of the two bodies to be quiescent, and the other to revolve about it.
- I Irregularities of the same kind take place among the primary planets by their mutual actions on each other, but the quantities are not considerable. Hence the apsidal of the planets are found to move (224, o) in consequentia, but so very slowly, that some have doubted whether they move at all.
- K The motions of the aphelia of Saturn, Jupiter, Mars, the Earth, Venus, and Mercury, as deduced from the comparison

comparison of distant observations, are respectively, $2^{\circ} 30'$, $1^{\circ} 43' 20''$, $1^{\circ} 51' 40''$, $1^{\circ} 49' 10''$, $4^{\circ} 10'$, $1^{\circ} 57' 40''$, in a century. The actions \mathbf{L} of the inferior planets on each other are very minute, on account of the smallness of their bulks; but those of Jupiter and Saturn are not altogether insensible. When Jupiter is between the Sun and \mathbf{M} Saturn, its whole attraction acts upon Saturn, and increases the gravity of that planet towards the Sun. This is found, by comparing the respective masses of Jupiter and the Sun, and the respective squares of their distances from Saturn to be equal to $\frac{1}{11}$ of the Sun's action upon Saturn. Saturn, \mathbf{N} on the other hand, at the conjunction, acts upon Jupiter and the Sun in the same direction, and therefore disturbs their relative position only so far as its actions on each are not equal. The difference of these actions is found by the same principles to be $\frac{1}{11}$ of Jupiter's whole gravity.

C H A P. IV.

OF THE FIGURES OF THE PLANETS; THE PRE-
CESSION OF THE EQUINOXES, AND THE NUTA-
TION OF THE EARTH'S AXIS.

- O A MASS of fluid matter will, by its gravity, form itself into a sphere. For if the whole mass be conceived to be divided into a number of similar pyramids or columns, terminating in the center of gravity, and one of these columns be longer or higher than the rest, its projecting part will spread sideways over the other columns, till the heights are all equal to its own. The same is true of any other eminences or longer columns. Therefore, when all the subsidences are effected, and the mass is at rest, its form will be that of a solid, whose surface is every where equidistant from its center. And this is the property of a sphere.
- P This takes place in a mass whose parts preserve the same situation with respect to its center; but if the sphere be caused to revolve on its own axis, a centrifugal force will be produced that will diminish the gravity of all its parts, except those which are situated in the axis of rotation. This diminution will be greatest in the equator, because the velocity is there greatest, and because the centrifugal force acts directly against the force of gravity. And the nearer any parallel of latitude or circle of rotation is to one of the poles, the less will

will the gravity of the parts be affected, both the above mentioned causes being less. The equilibrium, before subsisting between the columns in a spherical figure, will consequently be destroyed, and the same effect must take place, as would have followed if the columns at the polar regions had been lengthened or augmented in mass beyond those near the equatorial parts, that is to say, the columns near the poles will spread over those towards the equator, till the difference of their lengths compensates for the difference of their gravities. Thus Q the sphere, by its rotation, will be changed into a solid, whose radii, drawn to the center, are longest near the equator, and shortest towards the poles, the axis being the shortest of all its diameters.

By computation grounded on these and other R considerations, it is shewn, that bodies at the equator of the Earth lose more than $\frac{1}{289}$ part of their gravity, and that the equatorial diameter is to the axis as 231 to 230, upon the supposition that the Earth is every where of the same uniform density. For what has been said of a fluid mass will hold good of the Earth, since if it were not of this figure, but spherical, the ocean would overflow the regions near the equator, and leave the polar regions elevated many miles above the level of the sea. But experience shews, that the land is in general no more elevated above the sea in one part of the globe than another.

This decrease of gravity towards the equator T is remarkably seen in the motion of pendulums.

For

For a pendulum, which in a higher latitude vibrates seconds, is found to go slower at the equator, and that in a much greater proportion than can arise from the lengthening of the rod by heat, nay, even in the coldest parts of the mountains of Spanish America, which are constantly covered
 u with snow. From the just mentioned quantity of diminution of gravity, it is not difficult to compute the length of a pendulum (87, v) which shall vibrate seconds in a given latitude, and from the agreement of these computations with experience, the oblate spheroidal figure of the Earth, as also the diurnal rotation from which it originates, are both confirmed.

v The same conclusion has likewise been obtained from the labours of many ingenious and learned men, who have actually measured the lengths of certain portions of the meridian in different latitudes, by which it appears, that the degrees are shorter towards the equator than nearer the poles. Whence it follows, that the meridian is more curved near the equator, and less near the poles, or in other words, that the Earth is flattened about the polar regions.

w The measure of a degree of the meridian, beginning at the equator, was found to be 56750 French toises, and the measure of a degree of the meridian cutting the arctic circle, was found to be 57422 French toises *.

* See De la Lande's *Astronomie*, § 2655, & seq. for a detail of the principal enterprises on this interesting subject.

These mensurations constitute the experimental x proof of the Earth's rotation on its axis; for it is evident, that a centrifugal force cannot be produced but by an absolute motion: and as the effects of this force are observed in the figure of the Earth, and not at all in the heavens, the motion of the Earth must be absolute and real, and that of the heavens only relative and apparent.

The planet Jupiter revolves on its axis in less y than ten hours; a rapidity which much exceeds that of the Earth; and its figure differs accordingly much more from that of a sphere, its equatorial diameter exceeding its polar diameter, according to the observations of astronomers, as 13 to 14 (126).

It has also been already noticed, that a similar z phenomenon is seen in the planet Mars (126).

If a number of fluid bodies revolved about the A Earth at equal distances from its center, they would, by the action of the Sun, or any other planet, be subject to irregularities of the same kind as the Moon has been shewn to have in its motion; that is, they would approach nearer, and move swifter at the conjunction and opposition than at the quadratures (221, 1. 222, 1). And if the B number of bodies were so great as to become contiguous, and form a fluid ring or circle, the parts of this ring would be affected in the same manner. If it were inclined to the ecliptic, the nodes would be stationary when in the conjunction or opposition (226, u) and be carried in a retrograde direction

tion in the other revolutions (229, z), but most swiftly when they were situated in the quadratures (226, v). Its inclination would likewise vary in every revolution (230, B), and in a space of time somewhat less than a periodical year would be diminished and increased, by turns, twice (230, A. 231, F).

c Suppose this fluid ring to be of the same diameter as the Earth, to be placed in a cavity hollowed round the Earth at the equator, and to revolve in the same time and direction as the Earth does on its axis. Its motion would not then be uniform (237, B), but at the conjunction and opposition swifter than the surface of the Earth, and slower at the quadratures; consequently, with respect to the surface of the Earth, it would ebb and flow like a sea. For, by reason of the increased swiftness at the conjunction and opposition, and the retardation at the quadratures, the fluid, between the conjunction or opposition and the ensuing quadrature, would form a cumulus or heap, while a correspondent defect would happen in the other quadrants preceding the conjunction and opposition.

D If this ring be now supposed to be frozen or converted into a solid, the flux and reflux will cease, but the precession of the nodes and the libratory increase and decrease of the inclination will remain (237, B). Suppose the ring to adhere to the surface of the Earth at the equator, instead of being admitted into a cavity; it will then communicate part of its motion to the Earth, the nodes of
whose

whose equator will recede, but with a much slower motion than those of the ring would have receded, if it had not adhered to the Earth; and the obliquity or angle which the equator makes with the ecliptic, will be diminished and increased alternately twice in a year.

The elevation of the equatorial parts of the Earth have the same effect as such a ring would have; for the excess of matter in those regions supplies its place.

Astronomers begin the year in the Spring, when the Sun is in that node of the equator, or equinoctial point at which the days begin to lengthen in the northern hemisphere. Now it is plain (187), that if the equinoctial points had no motion, the Earth would complete one revolution in its orbit in the same time that the Sun employs in apparently passing from one of the equinoxes, and returning again to the same. But, because of the retrograde motion, the line of the nodes of the equator, or diameter of the Earth which joins the equinoctial points, is brought to coincide again with the line which joins the centers of the Sun and Earth, before its periodical revolution is completed; and therefore the circle of the seasons is performed in less time than the Earth's revolution in its orbit. The actions both of the Sun and Moon on the redundant matter in the equatorial regions tend to produce this motion, which is so slow, that a complete revolution will not be finished in less than twenty-five thousand years.

This

- H This is called the precession of the equinoxes, and is the reason that the fixed stars appear to advance in longitude about 50 seconds of measure in a year; whence it has happened, that since the time of Ptolemy, the zodiacal figures have advanced the greatest part of a whole sign: the constellation Aries being situate in that part of the ecliptic which is denominated from Taurus, Taurus in the place of Gemini, &c. The difference between the natural year or period of the seasons, and the periodical year, or time of the Earth's revolution in its orbit, is $26^m 34\frac{1}{2}^s$; for the natural year consists of $365^d 5^h 48' 45\frac{1}{2}''$, and the periodical year of $365^d 6^h 15' 20''$.
- K The sidereal year, or time employed by the Sun in returning to the same apparent position with respect to a fixed star, is $365^d 6^h 9' 1''\frac{1}{4}$. The difference between the periodical and sidereal year is occasioned by the motion of the apsis of the Earth's orbit (216, K).
- L The libratory variation of the inclination of the equator to the ecliptic is termed the nutation of the Earth's axis. The theory of attraction had ascertained its existence, long before astronomical observations were brought to a sufficient degree of perfection to render it sensible. Its whole effect scarcely amounts to 18 seconds. It was first observed by Dr. Bradley*.

* Phil. Trans. January, 1748.

C H A P. V.

OF THE TIDES.

THOUGH the cause of the tides may be collected from what was said in the last chapter; yet, as it is the only obvious instance we have of the mutual gravitation of the celestial bodies, it will be proper to give a more particular explanation of it.

If the Earth were every where covered with a deep sea, it is plain, from the reasons before recited (238, c), that the water would not, in the diurnal rotation, move with the same uniform velocity as the Earth. For, if the apparent diurnal revolution of the Moon be called a lunar day, and be divided into twenty-four equal parts or hours, the water situated near the meridian over which the Moon at any time is, will move swifter, and the water situated near the meridian six hours to the eastward or westward, will move slower: because the water on each parallel of latitude may be conceived to be a fluid ring, and will be affected by the disturbing force nearly in proportion to its diameter. The sea, then, being accelerated at the meridian upon which the Moon is, and retarded at the meridian, that is 90° or 6 hours to the eastward, will be accumulated between the two places; its greatest height being at the half distance,

distance, or meridian which the Moon has passed three hours. And on the other hand, the retardation at the quadrature to the westward, preventing the water from flowing as fast as the acceleration, at the meridian at which the Moon is, carries it away, the sea must of course be depressed between the two places, its greatest depression being at the half distance, or meridian at which the Moon will arrive in three hours. A similar accumulation and diminution will happen at the places which are diametrically opposite to those here described, though not altogether so great (232, o). The disturbing force of the Sun will act in a like manner, but less strongly; for, though the Moon's attractive force be vastly less than that of the Sun, yet, because its distance in comparison to that of the Sun from the Earth is very small, the forces with which it acts on different parts of the Earth will vary more considerably from parallelism and equality; and the irregularities in any system, which arise from the actions of forces from without, are occasioned (79, w), not by the whole actions of the forces, but only by their differences in quantity, or want of parallelism in direction.

Thus, it is evident, that the sea, as far as circumstances allow, must in every place be raised to its greatest and least height, alternately twice in each lunar day. Being elevated once at three lunar hours after the Moon has passed the meridian of the place, and once at twelve hours after, or three hours after the Moon has passed the opposite part
of

of the same meridian; and at six hours after each of these elevations its greatest depressions follow. This appears by the tides in the Atlantic ocean on the western coasts of Europe and Africa, and in the Pacific ocean on the open coasts of Asia and America, where high-water always happens about the third hour after the Moon has passed the meridian, except where the motion of the sea is somewhat impeded by flats or shoals.

The effects of the disturbing forces of the Sun and Moon are not seen distinctly, but compounding with each other produce a motion which is different from what would have arisen from the single action of either luminary. At the time of the conjunction or opposition their effects are united, and the tides are greatest, being what are called Spring-tides. When the Moon is in the quadrature, the Sun's action raises the water where the action of the Moon depresses it, and depresses it where the action of the Moon raises it: from the difference of their actions therefore arises the least, or, as they are called, Neap-tides. And, because the action of the Moon exceeds that of the Sun, high-water follows nearest the third lunar hour. At other times high-water arising from the lunar force would happen on the third lunar hour, and that which arises from the Sun's force on the third solar hour; but the forces being compounded, produce a tide which happens at some intermediate time, though always, on account of the greater force, nearest to the third lunar hour. Consequently, when the third solar w

hour precedes the third lunar hour, as is the case while the Moon is in the first and third quarters, high-water happens sooner than the third lunar hour, and the contrary happens when the third lunar hour precedes the third solar hour, as in the
x second and fourth quarters. It is to be noted, that no distinction is here made between the hour of the Sun or Moon's passing the meridian above the horizon, or beneath it; the effect being nearly the same with respect to the tides.

y The effects of the disturbing forces of the Sun and Moon depend likewise upon their respective distances from the Earth. For these effects are
z greater at less distances. And therefore in winter, when the Earth is in its perihelium, the Sun being nearer, causes the spring-tides to be somewhat greater and the neap-tides somewhat less, than in
a the summer; and the Moon being each month in its perigeum, does then, in like circumstances,
b cause greater tides than at other times; whence it happens, that if a great spring-tide happens when the Moon is in its perigeum, the next spring-tide will be less, because the Moon will be then in its apogeum, or greatest distance.

c The tides vary likewise in consequence of the
d varying declinations of the Sun and Moon. For if either of these luminaries were supposed to be at the pole, it would neither accelerate nor retard the diurnal rotation of the water, but would occasion a constant elevation at the poles, by diminishing the effect of gravity there, and a constant
stant

*stant depression at the equator, from an opposite cause (219, c. 220, 2). Therefore, no alternate rise and fall of the water, or tide, would be produced. Consequently, as the Sun and Moon decline towards the pole, they gradually lose their effects, and the tides become less considerable. When the Sun is in the equator, and the Moon at the tropic, or its greatest declination, the tides are less than when the Moon is at the equator, and the Sun in the tropic: because in the first case the Sun's influence is the greatest possible, and the Moon's least; and in the latter the Moon's influence is the greatest possible, and the Sun's least; and as the tides depend more upon the Moon's influence than that of the Sun, they are greatest when its action is greatest. When the Sun and Moon are both in the equator, the spring-tides are the greatest of any. However, because the Earth is nearer the Sun in winter than in summer, the greatest autumnal spring-tides are generally later than the equinox; and the greatest vernal spring-tides are generally before the equinox.

When the Moon declines either to the northward or southward of the equator, one of the greatest elevations of the water follows the Moon, and describes nearly the same parallel of latitude as the Moon, by the diurnal motion, apparently describes; and the other greatest elevation being diametrically opposite, must, of course, describe a parallel of latitude at an equal distance on the other side of the equator. The greatest elevation,

which moves on the same side of the equator with a given place, will come nearer to it than the opposite elevation; and therefore when the Moon declines towards the same side of the equator, as that on which the given place is situated, the day-tides, or tides which happen while the Moon is above the horizon, will be greatest, and the night-tides, or those which happen while the Moon is beneath the horizon, will be least. And the contrary happens when the Moon declines to the other side of the equator. Thus, the elevation at high-water is alternately greater and less; and the difference is greatest when the Sun and Moon both describe the same tropic, because the opposite elevations then describe the tropics, which are the farthest from each other of any two parallel circles they can possibly describe. This difference is found to be about a foot at Plymouth, and fifteen inches at Bristol.

If the actions of the Sun and Moon were to cease at once, the tides would not immediately cease, but would continue for some time by the undulating motion of the water. This undulation would be greater, if the actions were to cease at the time of a great tide than at the time of a less; and therefore less tides, which succeed greater, are more increased by it than greater tides which succeed less: consequently the difference between the tides is rendered less than it would otherwise have been, and the greatest spring and neap-tides do not happen precisely at the conjunction or opposition, and quadratures, but two or three tides later.

If the greatest acceleration and retardation of the diurnal motion (241, N) cannot subsist in the same sea at the same time, the accumulation or defect must consequently be less; that is to say, if one of the shores or coasts of any sea be less than ninety degrees to the eastward or westward of the other, and the eastern coast, for instance, be immediately under the Moon, the acceleration will, by causing the water to rise, occasion a defect or fall to the westward, because the western parts, being retarded, do not follow with a velocity sufficient to supply what is carried to the eastward by the acceleration: and the greater this retardation the greater the defect or fall. But since by the supposition the western shore is not 90 degrees distant, the retardation is not there so great as it would have been had the sea been wider; and therefore the fall in that sea is not so great. By a like argument it appears, that when the Moon is at the meridian of the western coast the elevation is less, if the sea be less than ninety degrees from east to west. Hence in small inland seas the tides are inconsiderable; and for this cause, in the Atlantic ocean the tides do not rise so high between the tropics as they do farther to the north or south, the sea being narrower between America and Africa in the lower than in the higher latitudes. From hence also follows the reason why the tides are so small as they are found to be at the islands of St. Helena and Ascension, which lie in the middle of that sea; for, since the water cannot rise on the shore but by falling at the other, it must continue at a mean height at these intermediate distant islands.

R This theory of the tides is perfectly consentaneous to experience in the open and deep oceans; and in the lesser seas, as has been observed, the tides are very small. But, when those less seas have a free communication with the ocean, the tide flows into them in a kind of wave, which on its arrival at
 S any place causes high-water. Thus it is high-water in the ocean to the westward of England and Ireland at the third lunar hour; after which it begins to subside. This subsidence must, of course, raise the water round about, whence a flood begins to enter the English channel at about the sixth hour, its course being retarded by the shallowness of the water. Another flood enters the German sea to the northward, near the Orkney islands, and proceeds to the southward. As these floods proceed on their respective courses, it is high-water successively at every place on the coasts at which they arrive, and when the wave has passed any place the water begins there to subside. For example; it is high-water at Plymouth about the sixth hour, at the Isle of Wight about the ninth hour, and at London-bridge about the fifteenth hour after the Moon has passed the meridian, and caused the tide in question. Therefore, when it is high-water at Plymouth the water out at sea has half subsided; when it is high-water at the Isle of Wight it is low water out at sea; and when it is high-water at London-bridge it is low water at the Isle of Wight, and a second flood or elevation has already come to its height out at sea.

There

There are situations where the tide may be carried to the same port by different passages or channels, and may pass quicker through one passage than another: in which case, the same tide, arriving at different times through the different passages, must occasion a variety of phenomena. Suppose two equal tides to arrive at the same port from different places; the one at the third, and the other at the ninth hour after the Moon has passed the meridian; the first tide therefore preceding the latter by six hours; and suppose the Moon to be at the equator: then, every six hours a tide will arrive, which, flowing in at the same time as the preceding equal tide ebbs out, will cause the water to continue at the same height, and thus it will neither rise nor fall during the whole day. But if the Moon decline from the equator, the tides in the ocean will be alternately greater and less, as has been observed; and therefore there will arrive at this port, alternately, two greater floods proceeding from the greater tide in the ocean, and two less floods proceeding from the lesser tide in the ocean. At the mean or intermediate time between the arrivals of the two greater tides, the water will then be highest; between a greater and a less tide it will be at a mean height; and lowest of all at the middle time between the arrivals of the two less tides. By these means, in the space of twenty-four hours, the sea will rise to its greatest, and fall to its least height but once, instead of twice, as in general it does in other places; and if the Moon decline towards the
same

same side of the equator, as that on which the port is situated, the two greater tides will arrive at the third and ninth hours, and the greatest elevation will be at the sixth hour, or at about the setting of the Moon: the least elevation will consequently happen between the two least tides, at the eighteenth hour, or about the rising of the Moon. And the same effects will take place when the Moon declines to the contrary side of the equator; but with this difference, that whereas high and low water happened then respectively at the setting and rising of the Moon, they will in the present case happen respectively at the rising and setting of the Moon.

w A remarkable instance of all these particulars is adduced by Dr. Halley, in the port of Batsha, in the kingdom of Tonquin, which lies in $20^{\circ} 50'$ north latitude. There, on the day on which the Moon passes the equator, the water stagnates; afterwards, on the Moon's declining to the northward, it begins to ebb and flow; not twice in the day, as in other ports, but once only; and high-water happens at the setting, and low-water at the rising of the Moon. The tides increase with the Moon's declination for seven or eight days; after which, for the next seven days, they decrease by the same gradation as they before increased, till the Moon's arrival again at the equator, when they cease, and upon its changing its declination are reversed. For while its declination becomes southerly, low-water happens at the setting, and high-water at the rising
of

of the Moon; which continues till its declination again changes. Now it appears, that the tide must come to this port by two inlets or passages; one between the continent of China and the island Luconia, communicating with the Chinese ocean, and the other between the island of Borneo and the continent. But whether the tide arriving from the Indian ocean, after a course of twelve hours, and from the Chinese ocean after a course of six, and thence happening on the third and ninth lunar hours, be the cause of these appearances; or whether some other circumstances may not be concerned in producing the effect, must be determined by observations on the neighbouring coasts.

B O O K II.

S E C T. I.

Of Light.

C H A P. I.

CONCERNING THE MOTION OF LIGHT.

IN the former part of this Treatise our attention x has been employed in considering such effects as arise from the motions and mutual actions of bodies, of magnitude sufficient to become the objects of our senses. It may be easily seen, that phenomena of this kind are but few compared with the very great number of operations which arise from the actions of bodies too minute to be determined but by deductions or inferences made from their effects. We have contemplated the great outline of the universe, and its vastness cannot but excite the astonishment of creatures who are destined at present to occupy an exceedingly small part of it. As we proceed to examine that small part, we shall develop a scene of another kind, which, though expanded through a less portion of space, is equally immense and unlimited with regard to the field it affords for admiration and perpetual discovery.

The success of every enquiry depends in no small y degree on the order employed in the several re-

searches to be made. It is obvious, that the probability of error is greater the more complicated the subject; whence the necessity of first examining the most simple, and thence proceeding to more compounded objects, is evident. This principle leads us, in our consideration of the particular properties of various bodies, to attend first to those of light.

- z It is generally agreed, that light consists of small bodies or particles, projected with great velocity in all directions, from the luminous or radiant body. No solid objections have been made to this hypothesis, which appears to be more simple than any other, and is perfectly consistent with all the phenomena yet observed; and these are so many and so various as to leave very little doubt of its truth.
- A The velocity of light was first determined by Monf. Romer, from observations on Jupiter's Moons (140, w) and the measure deduced from his observations was afterwards confirmed and established by the discovery of the aberration of the fixed stars. The principles on which this discovery is founded may be familiarly explained as follows *.
- B Suppose a tube to be erected perpendicular to the horizon, at a time when it rains, the drops falling perpendicularly down, and suppose the diameter of the tube to be such as to admit but one drop at a time: then it is plain, that if a

* The original account of this discovery may be seen in a paper by its inventor Dr. Bradley, inserted in the *Philosophical Transactions* for the year 1728, No. 406.

drop of water enter the orifice of the tube it will fall to the bottom without touching its sides. But if the tube, without altering its perpendicularity, be moved along in the direction of the horizon, any drop that enters will strike against one of its sides, and none will pass clear through while the motion continues, unless the upper end of the tube be also inclined towards the part to which its motion is directed.

Thus, if AB (fig. 70) represent the horizon, CD the perpendicular tube, and GD the course of a drop of rain: then, if CD be moved towards A , while the drop is falling within the tube, it is evident that the inner surface of the tube, which is situated towards B , will be carried against the drop, and prevent its arriving at the bottom without touching. But if the inclined tube EC be moved with a similar motion to that of the drop from E to D , in the time that the drop moves from C to D , the lower orifice of the tube and the drop will be found at D at the same instant; and the velocity of the drop will be expressed by CD , and that of the tube by ED .

The same reasoning holds good, if instead of drops of rain we suppose particles of light, and a telescope instead of a tube. For to an observer, who through the tube CD views the vastly distant object G , if the motion of light be instantaneous, or infinitely swift, no finite motion of CD , its position being unaltered, can prevent its being visible; since, by the supposition, the light which enters at C will

It will arrive at D before C D can have moved at all. But if light be propagated in time, and the observer be carried by a motion similar, as to acceleration, to that of light, the tube must be inclined to the ray in an angle, whose sine is to the sine of C E D, or the angle the tube makes with the line of the observer's motion, as the velocity of the observer is to the velocity of light. For in the triangle D C E, the sides D E and D C, which express these velocities, are as the sines of their opposite angles. Hence if the angle of the inclination of the tube to the ray of light, together with the velocity and direction of the observer's motion be known, the velocity of light may be determined.

- E By this theory, which is established by a great number of observations on stars of different magnitudes and situations, it appears, that the small apparent motion the fixed stars have about their real places, which is called their Aberration, arises from the proportion which the velocity of the Earth's motion in her orbit bears to that of light. This proportion is found to be as 10210 to 1: from whence it follows, that light moves or is propagated as far as from the Sun to the Earth in 8' 12". And it likewise appears, that the velocity of light is uniform, and the same, whether original, as from the stars, or reflected, as from the satellites of Jupiter (140, w).
- H The velocity of light being known, an estimate might be made of the magnitude of its particles, if we were in possession of good observations

tions of the effects of their momentum. For example, it is found, that a ball from a cannon at its first discharge flies with a velocity of about a mile in * eight seconds, and would therefore arrive at the Sun in thirty-two years, supposing it to move with unremitted velocity. And light, as was before observed, moves through that space in about eight minutes, which is two million times as fast. But the force with which bodies move are as their masses multiplied by their velocities (19, L); and consequently if the particles of light were equal in mass to the two millionth part of a grain of sand, we should be no more able to endure their impulse than that of sand shot point blank from the mouth of a cannon.

From several experiments † that well deserve L to be repeated, in which the Sun's rays were thrown, much condensed, upon a very light lever, suspended so as to be removeable horizontally, it seems probable that the momentum of light may be great enough to be rendered sensible by the velocity it communicates to bodies by its impulse.

The rarity of light is not less a matter of wonder M at the first consideration, than its velocity and the minuteness of its particles. For its rays cross each other in all directions without the least apparent disturbance. We can easily see through a small

* This varies according to the charge of powder and other circumstances.

† By Mr. Michell. See Priestley's Optics, p. 387.

hole, not exceeding the $\frac{1}{16}$ part of an inch, the objects, as the sky, trees, houses, &c. which occupy almost an entire hemisphere. The light proceeding from all these objects must therefore pass at the same time through the hole in a very great variety of directions before they arrive at the eye; yet it does not appear that vision is in the least disturbed by that means.

N This is, however, explained without difficulty from a well known circumstance relative to the organs of vision. For the action of light produces an effect on the eye that is not instantaneous, but lasts a considerable time. Suppose the effect of light on the eye to continue without sensible diminution, after the light has ceased to act, for the $\frac{1}{16}$ part of a second, and it will follow that a succession of particles of light arriving at the eye, by equal intervals, to the number of three hundred in a second, will be sufficient to excite a constant and uniform sensation of the presence of light. And since the velocity of light is such that it passes through about one hundred and seventy thousand geographical miles in a second (140, w); this space divided by 300, will give nearly 570 miles for the distance between each of the above-mentioned successive particles. It is not therefore to be wondered, that the particles of light do not interrupt each other, when we attend to their extreme minuteness (257, κ) and the very great distance at which they may follow each other without preventing the constancy of their effect.

That

That the effect of light on the eye remains for a time is evinced from several observations. We are continually shutting our eyes, or winking, and should on those occasions lose sight of all the surrounding objects, if the effect of their light did not continue during the time the eye is shut. Again, if a stick, or any other object, be whirled round in a circle, there is a certain velocity beyond which the object will fill the whole circle. This experiment is vulgarly made with a lighted firebrand, and obviously shews that the impression, made on the eye by the light of the firebrand, when in any given point of the circle, is sufficiently lasting to remain till the firebrand has described the whole circle, and again renews its effect, which is by that means rendered continual and uniform. Every one must have been sensible of the strong and lasting impression the Sun's direct light makes upon the eye; and impressions of the same kind from other objects, though weaker, are much less so than is generally imagined.

With respect to the duration of the impressions of light, it has been observed, that the teeth of a cog-wheel in a clock * were still visible in succession when the velocity of rotation brought 246 teeth through a given fixed point in a second. In this case it is clear, that if the impression made on the eye by the light reflected from any tooth, had lasted without sensible diminution for the 246th part of a second, the teeth would have formed one

* Watfon on Time.

unbroken line, because a new tooth would have continually arrived in the place of the anterior one before its image could have disappeared. If a live coal be whirled round, it is observed*, that the luminous circle is complete when the rotation is performed in $\frac{8}{100}$ of a second. In this instance we see that the impression was much more durable than the former. Lastly, if an observer sitting in a room direct his sight through a window, to any particular object out of doors, for about half a minute, and then shut his eyes, and cover them with his hands, he will still continue to see the window, together with the outline of the terrestrial objects bordering on the sky. This appearance will remain for near a minute, though occasionally vanishing and changing color, in a manner that brevity forbids our minutely describing. From these facts we are authorized to conclude, that all impressions of light on the eye last a considerable time; that the brightest objects make the most lasting impressions; and that, if the object be very bright, or the eye weak, the impression may remain for a time so strong, as to mix with and confuse the subsequent impressions made by other objects. In the last case the eye is said to be dazzled by the light.

Q The space through which light passes is called a medium; by which term reference is had to the quantity or density of the matter contained in the

* By M. D'Arcy. See Mem. de l'Acad. des Sciences, 1765; or Priestley's Optics, p. 634.

space: thus, glass and air are mediums, but a vacuum, or space absolutely void, is a medium also.

When light passes through mediums, either absolutely void, or containing matter of an uniform density, and of the same kind, it always is found to proceed in straight lines. Whence it follows, that the rarity of light increases as the square of the distance from the radiant body. For the light which falls on the square $ABCD$, (fig. 71) from the point R , at the distance RA , will be spread over a surface, $abcd$, four times as large at twice the distance, or RA .

C H A P. II.

OPTICAL DEFINITIONS AND PRINCIPLES.

WHEN a ray of light passes out of one medium into another, and is bent out of its course at their common surface, this bending is called refraction.

When a ray of light proceeds to the common surface of two mediums, and instead of passing from the one into the other, is turned back into the first, this turning back is called reflection.

The angle of incidence is the acute angle which the line described by the ray of light makes with a line drawn perpendicular to the surface at the point of incidence.

The angle of reflection or refraction is the acute angle, which the line described by the ray of light

- after reflection or refraction makes with the perpendicular to the surface at the point of incidence.
- w Thus, if rs represent the common surface of two mediums, ac (fig. 72) a ray of light incident at c , and pq a line intersecting the surface at right angles at c ; then the angle acp is the angle of incidence. If it be reflected at c , so as to return in the line cb , then the angle pcb is the angle of reflection: and if it be refracted at c , so as to proceed in the line cf , the angle qcf is the angle of refraction.
- x The angles of incidence, reflection, and refraction lie in one and the same plane.
- y The angle of reflection is equal to the angle of incidence.
- z If the refracted ray be returned directly back to the point of incidence, it shall be refracted into the line which was before described by the incident ray.
- a The refractive powers of different mediums are nearly as their densities; that is to say, if a ray of light pass out of a rarer into a denser medium, it will in general be refracted towards the perpendicular, so that the angle of refraction will be less than the angle of incidence. And the contrary will happen if the ray pass out of a denser into a rarer medium (262, z).
- b The sine of the angle of incidence is either accurately or very nearly in a given ratio to the sine of the angle of refraction, in all obliquities of the incident ray.

All objects seen by reflection or refraction appear in that place or direction, from whence or in which the rays were last reflected or refracted to the eye.

Thus, if the ray Ac (fig. 72) proceed from an object at A to c , and be thence reflected to the eye of the spectator at B , the object will be seen not at A but at T , in the direction of the reflected ray Bc . And if the ray Fc proceed from an object at F , and be refracted into the direction cA to the eye of a spectator at A , the object will be seen not at F but at N , in the direction of the refracted ray Ac .

On this account it is that objects are seen in mirrors or looking-glasses, and that objects seen under water appear out of their true places. Let $ABCD$ (fig. 73) represent a vessel containing water, whose surface is FG , and let o represent an object at the bottom. Then, to an eye at E the object o will be seen at K , by means of the ray OL , which passing from a denser to a rarer medium (262, A) is refracted from the perpendicular PQ into the direction LE . Or let $ABCD$ (fig. 74) represent a vessel so placed with respect to the candle E , that the shadow of the side Ac may fall at D . Suppose it now filled with water, and the shadow will withdraw to d , the ray of light EA , instead of proceeding to D , being refracted to d . And there is no doubt but that an eye placed at d would see the candle at e in the direction of the refracted ray dA .

The foregoing principles are founded on experiment or observation, and the mathematical application of them to the rays of light which pass through glasses, or are reflected from mirrors of various figures, constitutes that branch of the science of optics which teaches the construction of instruments. But it will be proper to avoid the explanation of these for the present, till an account has been given of the various reflexivity, refrangibility, and colours of light.

C H A P. III.

OF THE VARIOUS REFRACTIBILITY OF THE RAYS OF LIGHT.

- G** LIGHTS which differ in colour differ also in refrangibility, and the contrary.
- H** Let AB (fig. 75) represent a wedge or triangular prism of glass, then the triangle ABC (fig. 76) may be conceived to be a section of the same, at right angles to its axis. Suppose JN to be a ray of light incident at N , and thence refracted to E , on the surface CB , where it is again refracted into the direction EM ; suppose in to be another ray parallel to the former, and consequently incident at n , with the same angle. Now, if the ray in have exactly the same capability or disposition to be refracted by the prism, as the ray JN , the angles of refraction will also be equal, and in will,

will, when refracted into the directions ne and em , still continue parallel to the ray jN , which is refracted into nE and EM . But if it be more refrangible it will be refracted into directions, as nf and fg , verging more towards the base AC , or, if less refrangible, it will be refracted into directions, as nh and hk , that verge less towards the base AC . Whence it appears, that if a pencil or collection of rays fall parallel to each other on one of the sides of a prism, and do not proceed parallel to each other on their emergence, it must be because some of the rays are more refrangible than others.

Let the space contained between EG and MA (fig. 77) represent a darkened chamber, of which those lines represent the sides. Let jN represent a pencil of light from the Sun, passing through the hole F , and incident on the side BC of the prism ABC . It is plain, that if the prism were not interposed, the pencil jN would proceed to s , and there illuminate a small circular spot on the wall; and from the preceding explanation it is evident, that if all the rays of light be equally refracted by the prism, the whole pencil, being equally turned out of its course, will suffer no alteration with respect to the parallelism of its rays, and consequently will, after refraction, proceed to Q , and there illuminate a spot similar to that which would have appeared at s . But if the pencil be composed of rays not alike refrangible, the most refrangible rays will be thrown farther from s , and the

the least refrangible, being less deflected out of their course, must fall on a part of the wall nearer to *s*, while those which are refrangible in the intermediate degrees will fall at interposed distances *x* between them. The actual experiment determines, that the Sun's light is composed of rays, whose refrangibilities are not all the same; for after emerging from the prism, instead of illuminating a circular space, they are spread into a long spectrum, bounded by right-lined sides and circular ends, and whose length is at right angles to the direction of the axis of the prism.

x This oblong spectrum is variously coloured. The lower part, which consists of the least refrangible rays, is of a lively red, which, higher up by insensible gradations becomes an orange; the orange in like manner is succeeded by a yellow; the yellow by a green; the green by a blue; after which follows a deep blue or Indigo; and lastly, a faint violet. With a prism, whose refracting angle was $63\frac{1}{2}$ degrees, so placed, that the angle of incidence on the first surface was equal to the angle of refraction at the emergence or second surface, the spectrum, received on a wall at the distance of $18\frac{1}{2}$ feet, was 10 inches, or $10\frac{1}{4}$ in length. Its breadth is in all cases equal to that of the circle, which would have been formed at that distance by the admitted beam or pencil of light, if the prism had not been interposed.

m The spaces occupied by the several colours of the spectrum answer to the subdivisions of a musical

fical chord: thus, if $AGMF$ (fig. 79) represent the spectrum, and the lines FM , ba , dg , zy , &c. mark the confines of the colours; the space $ma b f$ being occupied by the red, $agdb$ by the orange, $gyzd$ by the yellow, and so forth, in the order above-mentioned, and GM be prolonged to x , so that mx may be equal to GM , the lines Gx , lx , ix , ex , yx , gx , ax , mx , will be in proportion to each other as the numbers $1, \frac{8}{9}, \frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{2}{7}, \frac{1}{2}$, and therefore express the chords of the key, tone, less third, fourth, fifth, greater sixth, greater seventh, and octave. Or, to render it more familiar to those whose knowledge of music is merely practical, let AB represent the string of a violin or guitar, and a fret or small bridge be fixed on the finger-board at the middle distance a , between A and B ; and likewise frets at g , f , e , d , c , b , so that the distances ag , gf , fe , ed , dc , cb , and bA , may be in proportion to the spaces occupied respectively by the red, orange, yellow, green, blue, indigo, and violet, in the spectrum; then if the open string sound A , the regular ascent of the stopt notes will be in the minor third from that key; the notes being A , B , C , D , E , F sharp, G sharp, and A octave.

Those rays which have the same degree of refrangibility will, after refraction, fall within a circle equal to that which would have been illuminated by the light if suffered to proceed to s (264, H): and therefore the spectrum may be conceived to be composed of an indefinitely great number

- number of such equal circles, whose centers are all on the same line. For example; if $A B D C$ (fig. 78) represent the spectrum, the circle $A B$ being formed by the red, or least refrangible rays, and the circle $D C$ by the violet, or most refrangible rays, then the rays, whose refrangibility is intermediate, will form an innumerable series of circles, and fill up the whole space, so that $A C$ and $B D$ will appear as right
- o lines. Now, it is observable, that though the light in the spectrum thus separated into its original rays is much less compounded than before, yet it is still compounded in no small degree by the interference of the circles with each other, particularly at the line $E F$, equidistant between $A C$ and $B D$; and that at the lines $A C$ and $B D$, where the circles do not interfere at all, the light is perfectly homogeneous
- P or uncompounded. But because the colours in the spectrum contiguous on either side of any given colour do by mixture compound a colour that differs insensibly from the original intermediate colour itself, a right line drawn perpendicularly across the spectrum will be found in the same colour
- Q throughout. For most experiments in which uncompounded light is required, that of the spectrum will be found sufficiently so, but in cases where a greater nicety is demanded, the common diameter of the circles, or breadth of the spectrum, may be diminished by making the hole at F smaller, or the figure of the hole may be altered..
- R It is evident from what has been already said, that
- this phenomenon arises from the nature of light itself,
- some

some of the rays of which are more refrangible than others. And as an additional confirmation it is observed, that if the spectrum be received on a board which is perforated, so as to let pass one ray of light, or colour, that ray will not be changed by any refraction it may be afterwards made to suffer, but continues the same both in colour and refrangibility. And if the colours of the spectrum be by any reflection or refraction made to unite again, they will again form the compounded colour of whiteness.

The quantity of the dispersion of the rays of light, which at equal distances from the prism is nearly expressed by the length of the spectrum, does not follow the quantity of the refraction of the mean ray, except in mediums of the same kind. Thus, if two prisms of different kinds of glass re- fract the solar ray equally out of its first direction, the spectrum of colours formed by the one will be much longer than that formed by the other; and it is found, that in equal angles of mean refraction, w glass, in the composition of which much lead enters, disperses the light into its component colours much more than glass which abounds with alkaline salts*.

If by means of two prisms, a small piece of pa- per be illuminated, the one half with red, and the other half with violet light, and an observer view the same through another prism, the paper will, by the different refrangibility of the rays, appear di-

* *Acta* (or perhaps *Miscellanea*) *Berolinensia*, for 1766; quoted by Priestley in his *Optics*, p. 474.

vided into two. For the violet half being seen by a more refrangible light, will appear to be carried farther from its true place than the red, and will y therefore seem to be separated from it. The same is likewise true of colours which arise from the separation of light which is made by bodies on which it falls, and which we are apt to call natural colours; for if a paper be painted, the one half with a lively red, and the other half with an indigo, and it be placed in the Sun's light, it will in the same manner appear divided, if viewed through a prism.

C H A P. IV.

OF THE VARIOUS REFLEXIBILITY OF THE RAYS OF LIGHT.

z THE Sun's light consists of rays which differ in reflexibility, and those rays which are more refrangible are also more reflexible than others. Let A B C (fig. 80) represent a prism, whose angle B is a right angle, and the two angles A and C equal to each other. Suppose J N to be a beam of light which passes through the surface B C, and is incident on A C at N. It will then emerge in the direction N O, so that the sine of the angle of refraction O N W may be in a certain ratio, (262, B) to the sine of the angle of incidence B N Z, which in glass is as 3 to 2, nearly. Now, when the angle of incidence at N is such, that the sine of the angle of refraction is equal to the radius, the angle of refraction

fraction becoming a right angle, the ray cannot emerge, but will be totally reflected or turned back into the glass. This happens in glass when the angle of incidence is about 41° degrees.

That the component rays of light are not at all equally disposed to be reflected, is proved by turning the prism slowly on its axis, till the light begins to be reflected; for it then appears, that the more refrangible rays are reflected sooner, or at less angles of incidence than those which are less refrangible. Let nm represent the reflected beam, and suppose the prism vxy placed so as to receive and separate it into its component colours by refraction: then the light which first begins to be reflected, consisting almost intirely of violet rays, will by the second prism be refracted so as to fall at p , and paint a violet colour. As the first prism continues to be turned on its axis, the light is more and more copiously reflected, and the colours between p and t appear in succession according to their order in refrangibility; violet, indigo, blue, green, yellow, orange, and lastly red, at which time the reflection becomes total: the colours formed by refraction at hg disappearing as those at pt appear.

White light being proved to consist of rays which differ in refrangibility, reflexivity, and the power of exciting the idea of colour; it is clear that nothing more is necessary to account for the colours of bodies than to suppose each body endued with a power or aptitude to reflect the rays of one particular colour, and to imbibe the rest.

But

But the truth of this does not rest on mere supposition. Bodies exposed to the uncompounded light of the spectrum, are ever found to be of the colour of the light in which they are placed, with this only difference, that they appear much more lively in that colour, which is the same with that which they exhibit in the day light. And from hence it appears, that the colours of bodies cannot be so homogeneous and full as those of the spectrum; for since they reflect all colours in some degree as well as the principal or predominant one, that principal colour must be much diluted and weakened by the mixture. It may likewise from hence be inferred, that as the uncompounded colours are not changeable by refraction, so neither are they changeable by reflection.

Language being invented chiefly for the expression of ordinary events that do not require any great precision, it very frequently happens, that the same word is used to denote very different things. It is proper to be remarked, that the word colour is thus used. If the word be used to denote the sensation or idea excited in the mind, it is sufficiently obvious, that it cannot be scientifically used to denote that attribute by which bodies are able, by reflecting the rays of light, to produce the sensation. And still less ought it to be used to imply that quality the various kinds of light possess, of producing the sensation, when separated from each other, either by reflection from bodies or otherwise. It may, however, be allowed to use the

the terms coloured rays, or coloured bodies, though the sensation of colour, the specific properties of the rays, and of the reflecting bodies, are undoubtedly things very different from each other. So different, indeed, that this remark might with justice be supposed unnecessary, if experience had not shewn, that among the pretenders to philosophical knowledge some have been found capable of mistaking in this very particular *.

C H A P. V.

CONCERNING THE RAINBOW.

THE instance of the separation of the primary colours of light which seems most remarkable, is that of the rainbow. It is formed in general by the reflection of the rays of the Sun's light from the drops of falling rain, though frequently it appears among the waves of the sea, whose heads or tops are blown by the wind into small drops, and is sometimes seen on the ground when the Sun shines on a very thick dew. Cascades and fountains, whose waters are in their fall divided into drops, exhibit rainbows to a spectator, if properly situated during the time of the Sun's shining; and water blown violently out of the mouth of an observer, whose back is turned towards the Sun, never fails to produce the same phenomenon. This appearance is also

* The opposers of Newton's discoveries on light and colours have falsely affirmed, that he taught that the rays of light were coloured.

seen by moonlight, though seldom vivid enough to render the colours distinguishable; and the artificial rainbow may be produced even by candle-light on the water which is ejected by a small fountain or jet d'eau. All these are of the same nature, and dependant on the same causes, an idea of which may be formed by the following considerations.

M Let the circle $wqgb$ (fig. 81) represent a globe or drop of water upon which a beam of parallel light falls, of which let tb represent a ray falling perpendicularly at b , and which by consequence (262, γ , b) either passes through without refraction, or is reflected directly back from q . Suppose another ray ik , incident at k , at a distance from b , and it will be refracted according to a certain ratio (262, b) of the sines of incidence and refraction to each other, which in rain water is as 529 to 396, to a point L , whence it will be in part transmitted in the direction Lz , and in part reflected to m , where it will again in part be reflected, and in part transmitted in the direction mp , being inclined to the line described by the incident ray in the angle iof . Another ray an , still farther from b , and consequently incident under a greater angle, will be refracted to a point r , yet farther from q , whence it will be in part reflected to g , from which place it will in part emerge, forming an angle axr with the incident an , greater than that which was formed between the ray mp and its incident ray.

And

And thus, while the angle of incidence or distance of the point of incidence from B increases, the distance between the point of reflection and Q , and the angle formed between the incident and emergent reflected rays will also increase; that is to say, as far as it depends on the distance from B : but as the refraction of the ray tends to carry the point of reflection towards Q , and to diminish the angle formed between the incident and emergent reflected ray, and that the more the greater the distance of the point of incidence from B , there will be a certain point of incidence between B and w , with which the greatest possible distance between the point of reflection and Q , and the greatest possible angle between the incident and emergent reflected ray will correspond. So that a ray incident nearer to B shall, at its emergence after reflection, form a less angle with the incident, by reason of its more direct reflection from a point nearer to Q ; and a ray incident nearer to w , shall at its emergence form a less angle with the incident, by reason of the greater quantity of the angles of refraction at its incidence and emergence. The rays which fall for a considerable space in the vicinity of that point of incidence with which the greatest angle of emergence corresponds, will, after emerging, form an angle with the incident rays differing insensibly from that greatest angle, and consequently will proceed nearly parallel to each other; and those rays which fall at a distance from that point will emerge at various angles, and consequently will diverge. Now, to a spectator, whose back is turned

towards the radiant body, and whose eye is at a considerable distance from the globe or drop, the divergent light will be scarcely, if at all perceptible; but if the globe be so situated, that those rays which emerge parallel to each other, or at the greatest possible angle with the incident, may arrive at the eye of the spectator, he will, by means of those rays, behold it nearly with the same splendor at any distance.

N In like manner, those rays which fall parallel on a globe, and are emitted after two reflections, suppose at the points F and G, will emerge, at H, parallel to each other, when the angle they make with the incident, $\angle N$, is the least possible; and the globe must be seen very resplendent, when its position is such, that those parallel rays fall on the eye of the spectator.

O The quantities of these angles are determined by calculation, the proportion of the sines of incidence and refraction to each other being known. And this proportion being different (264, o) in rays which produce different colours, the angles must vary in each. Thus it is found, that the greatest angle in rain-water for the least refrangible, or red rays, emitted parallel after one reflection is $42^{\circ} 2'$, and for the most refrangible or violet rays emitted parallel after one reflection $40^{\circ} 17'$; likewise, after two reflections the least refrangible or red rays will be emitted nearly parallel under an angle of $50^{\circ} 57'$, and the most refrangible or violet under an angle of $54^{\circ} 7'$; and the intermediate colours will

will be emitted nearly parallel at intermediate angles.

Suppose now, that o (fig. 82) is the spectator's eye, and op a line drawn parallel to the Sun's rays, and let poE , poF , poG , poH , be angles of $40^\circ 17'$, $42^\circ 2'$, 50° , $57'$, and $54^\circ 7'$ respectively, and these angles turned about their common side op , will, with their other sides oE , oF ; oG , oH describe the verges of two rainbows as in the figure. For, if E , F , G , H be drops placed any where in the conical superficies described by oE , oF , oG , oH , and be illuminated by the Sun's rays sE , sF , sG , sH ; the angle sEo being equal to the angle poE , or $40^\circ 17'$, will be the greatest angle in which the most refrangible rays can, after one reflection, be refracted to the eye, and therefore all the drops in the line oE must send the most refrangible rays most copiously to the eye, and thereby strike the sense with the deepest violet colour in that region. And in like manner the angle sFo being equal to the angle poF , or $42^\circ 2'$, will be the greatest in which the least refrangible rays after one reflection can emerge out of the drops, and therefore those rays must come most copiously to the eye from the drops in the line oF , and strike the sense with the deepest red colour in that region. And, by the same argument, the rays which have the intermediate degrees of refrangibility will come most copiously from drops between E and F , and strike the senses with the intermediate colours in the order which their degrees of refrangibility require;

that is, in the progress from *E* to *F*, or from the inside of the bow to the outside, in this order, violet, indigo, blue, green, yellow, orange, red. But the violet, by mixture of the white light of the clouds, will appear faint, and inclined to purple.

Q Again, the angle sgo being equal to the angle pog , or $50^{\circ} 57'$, will be the least angle in which the least refrangible rays can, after two reflections, emerge out of the drops, and therefore the least refrangible rays must come most copiously to the eye from the drops in the line og , and strike the sense with the deepest red in that region. And the angle sho being equal to the angle pon , or $54^{\circ} 7'$, will be the least angle in which the most refrangible rays, after two reflections, can emerge out of the drops, and therefore those rays must come most copiously to the eye from the drops in the line oh , and strike the sense with the deepest violet in that region. And, by the same argument, the drops in the regions between *G* and *H* will strike the sense with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progress from *G* to *H*, or from the inside of the bow to the outside in this order, red, orange, yellow, green, blue, indigo, and violet. And since the four lines oe , of , og , oh may be situated any where in the above-mentioned conical superficies, what is said of the drops and colours in these lines is to be understood of the drops and colours every where in those superficies.

R Thus there will be made two bows of colours,

an

an interior and stronger, by one reflection in the drops, and an exterior and fainter by two; for the light becomes fainter by every reflection; and their colours will lie in a contrary order to each other, the red of both bows bordering upon the space GF , which is between the bows. The breadth of the interior bow, EOF , measured cross the colours, will be $1^{\circ} 45'$, and the breadth of the exterior, GOH , will be $3^{\circ} 10'$, and the distance between them GOF , will be $8^{\circ} 55'$, the greatest semidiameter of the innermost, that is, the angle POF , being $42^{\circ} 2'$, and the least semidiameter of the outermost POG being $50^{\circ} 57'$. These are the measures of the bows, as they would be, were the Sun but a point; for, by the breadth of its body, the breadth of the bows will be increased, and their distance diminished by half a degree, and so the breadth of the interior iris will be $2^{\circ} 15'$, that of the exterior $3^{\circ} 40'$, their distance $8^{\circ} 25'$; the greatest semidiameter of the interior bow $42^{\circ} 17'$, and the least of the exterior $50^{\circ} 42'$. And such are the dimensions of the bows in the heavens found to be very nearly, when their colours appear strong and perfect.

The light which comes through drops of rain by two refractions without any reflection ought to appear strongest at the distance of about 26 degrees from the Sun, and to decay gradually both ways as the distance from the Sun increases and decreases. And the same is to be understood of light transmitted through spherical hail-stones. And if the

halo be a little flattened, as it often is, the light transmitted may grow so strong at a little less distance than that of 26 degrees, as to form a halo about the Sun and Moon; which halo, as often as the stones are duly figured, may be coloured, and then it must be red within, by the least refrangible rays, and blue without, by the most refrangible ones.

- T The light which passes through a drop of rain after two refractions, and three or more reflections, is scarcely strong enough to cause a sensible bow.

C H A P. VI.

OF THE SEPARATION OF THE ORIGINAL RAYS OF LIGHT BY REFLECTION OR TRANSMISSION, THAT DEPENDS ON THE THICKNESS OF THE MEDIUM UPON WHICH THEY ARE INCIDENT.

- U THE original or component rays of light are separable from each other, not only by refraction, or by varying the angle of incidence on a reflecting surface, but are likewise at like incidences more or less reflexible, according to the thickness or distance between the two surfaces of the medium on which they fall. They are also liable to be turned out of their direct course by approaching within a certain distance from a body, by which means a separation ensues, the rays being more or less deflected as they differ

differ in colour. Of these circumstances it will be proper to give some account.

If a glass or lens, whose surface is convex, or a portion of a sphere, be laid upon another plain glass, it is evident that it will rest or touch at one particular point only; and therefore, that at all other places between the adjacent surfaces will be interposed a thin plate of air, the thickness of which will increase in a certain ratio, according to the distance from the point of contact; that is to say, in arcs whose versed sines are very small, as the diameter of the sphere is to the sine of the arc, so is that sine to the versed sine or thickness of the air at the distance measured by the sine.

Light incident upon such a plate of air is disposed to be transmitted or reflected according to its thickness: thus, at the center of contact, the light is transmitted, and a black circular spot appears; this spot is environed by a circle, the colours of which, reckoning from the internal part, are blue, white, yellow, red; then follows another circular series, viz. violet, blue, green, yellow, red; then purple, blue, green, yellow, red; green, red; greenish blue, red; greenish blue, pale red; greenish blue, reddish white.

These are the colours which appear by reflection: by the transmitted light the following series are seen. At the centre white, then yellowish red, black; violet, blue, white, yellow, red; violet, blue, green, yellow, red, &c. so that the transmitted light at any thickness, instead of white, appears

- appears of the compounded colour which it ought to have after the subtraction of some of the constituent colours by reflection; after which series the colours become too faint and dilute to be discerned.
- z It is observable, that the glasses will not come into contact without a considerable degree of pressure.
- A By admeasurement it appears, that the rays of any particular colour are disposed to be reflected when the thicknesses of the plate of air are as the numbers 1. 3. 5. 9. 11. &c. and that the same rays are disposed to be transmitted at the intermediate thicknesses which are as the numbers 0. 2. 4. 6. 8. 10. &c.
- B The places of reflection or transmission of the several colours in a series are so near each other, that the colours dilute each other by mixture, whence the number of series in the open day-light seldom exceeds seven or eight: but if the system be viewed through a prism, by which means the rings of various colours are separated according to their refrangibilities, they may be seen on that side towards which the refraction is made, so numerous,
- c that it is impossible to count them. Or, if in a dark chamber the Sun's light be separated into its original rays by a prism, and a ray of one uncompounded colour be received upon the two glasses heretofore described, the number of circles will become very numerous, and both the reflected and transmitted light will remain of the same colour as the original incident ray. In this experiment it also is seen, that in any series, the circles formed by

by the less refrangible rays exceed in magnitude those which are formed by the more refrangible rays, and consequently that in any series the more refrangible rays are reflected at less thicknesses than those which are less refrangible.

If the light be incident obliquely, the rings of colours dilate and enlarge themselves; whence it follows, that the thickness required to reflect the colours of any series is different in different obliquities.

Water, applied to the edges of the glasses, is attracted between them, and filling all the intercedent space, becomes a thin plate of the same dimensions as that which before was constituted of air. In this case the rings become much fainter, but vary not in their species, and are contracted in diameter nearly in the proportion of 7 to 8: consequently the intervals of the glasses at like circles caused by these two mediums, water and air, are as about 3 to 4, that is, nearly as the sines which measure the angles of incidence and refraction, made at a common surface between them. And hence it may be suspected, that if any other medium, more or less dense than water, be compressed between the two glasses, their intervals at the rings caused thereby will be to the intervals at which similar rings are caused by the interjacent air, as the sine which measures the refraction made out of air, into that medium is to the sine of the incidence on the common surface.

These are some of the phenomena of light incident

dent on mediums which are environed by mediums of greater density, as air or water compressed or included between plates of glass. The same appearances follow, though with some little variation, when the colorific medium is denser than that in which it is inclosed.

- 1 It is well known that bubbles blown in soap-water exhibit a great variety of colours; but as these colours are commonly too much agitated by the external air to admit of any certain observation, it is necessary that the bubble be covered with a clear glass; in which situation the following appearances ensue: the colours emerge from the vertex or top of the bubble, and as it grows thinner by the subsidence of the water, they dilate into circles or rings parallel to the horizon, which slowly descend and vanish successively at the bottom. This emergence continues till the water at the vertex becomes too thin to reflect the light, at which time a circular spot of an intense blackness appears at the top, which slowly dilates sometimes to three quarters of an inch in breadth before the bubble breaks. Reckoning from the black central spot, the reflected colours are the same in succession and quality as those produced by the aforementioned plate of air, and the appearance of the bubble, if viewed by transmitted light, is also similar to that of the plate of air in like circumstances.

- 2 If the colours be viewed with different obliquities,

ties, their place is changed, but not near so much as in the plate of air.

The end of a small glass tube or pipe being melted, by turning the flame of a candle or lamp upon it, by means of a blow-pipe, may be blown into a bubble of an extreme thickness. Such a bubble will exhibit colours of the same kind as the foregoing, but much more brisk and lively. From which, and the premised observations, it is concluded that a denser medium inclosed by one that is rarer exhibits more lively colours than those which are produced by a rarer medium included in one that is more dense. It is also observable, that the colours produced by reflection from, or transmission through, dense substances, are less subject to vary by change of the obliquity of the incident light than they are in substances that are more rare.

By wetting very thin plates of Moscovy glass, whose thinness occasion the like colours to appear, the colours become more faint and languid, especially if wetted on the surface opposite to the eye; but no variation of their species is produced: so that the thickness of any plate requisite to produce any colour, seems to depend only on the density of the plate, and not on that of the ambient medium: and hence, if the suspicion formerly urged be true (283. 6), may be known the thickness which thin plates of any transparent substance have at the place where a given colour in any series is produced. For,

As

As the sine of the angle of incidence at the common surface

Is to the sine of the angle of refraction out of the given medium into air,

So is the thickness of a plate of air which exhibits the given colour

To the thickness of the given plate.

- P. As lenses ground to a long radius are necessary to be used in these experiments, and such are not very common, it may be an acceptable piece of information for the learner to know, that their place may be well supplied by two pieces of plate-glass, or even common glass. If these be previously wiped, and then rubbed together, they will soon adhere with a considerable degree of force, and exhibit various ranges of colours, much broader than those obtained by lenses. One of the most remarkable circumstances attending this method of making the experiment is the facility with which the colours may be removed, or even made to disappear by heats too low to separate the glasses. It seems most probable, that the operation of heat consists in augmenting the distance between the surfaces. A touch of the finger immediately causes the irregular rings of colours to contract towards their center in the part touched.

C H A P. VII.

GENERAL INFERENCES RESPECTING THE DISPOSITION TO BE REFLECTED OR TRANSMITTED, INTO WHICH THE RAYS OF LIGHT ARE PUT, BY THE ACTION THAT DEPENDS ON THE THICKNESS OF THE MEDIUM UPON WHICH THEY ARE INCIDENT.

THE experiments or observations in the last chapter being maturely weighed and considered, indicate the following theorem or general proposition; namely,

Every ray of light in its passage through any refracting surface is put into a certain transient constitution or state, which in the progress of the ray returns at equal intervals, and disposes the ray, at every return, to be easily transmitted through the next refracting surface, and, between the returns, to be easily reflected by it.

For, by those observations it appears, that one and the same sort of rays, at equal angles of incidence on any thin transparent plate, is alternately reflected and transmitted for many successions; accordingly, as the thickness of the plate increases in arithmetical progression of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, &c. so that if the first reflection, or that which makes the first or innermost ring of colours, be made at the thickness 1, the rays shall be

be transmitted at the thicknesses 0, 2, 4, 6, 8, 10, 12, &c. and thereby make the central spot and rings of light which appear by transmission, and be reflected at the thicknesses 1, 3, 5, 7, 9, 11, &c. and thereby make the rings which appear by reflection. And this alternate reflection and transmission continues for a great number of vicissitudes, and by other observations, which for the sake of brevity are omitted, for many thousands, being propagated from one surface of a glass-plate to the other, though the thickness of the plate be a quarter of an inch or above: so that this alternation seems to be propagated from every refracting surface to all distances without end or limitation. And because the ray is disposed to reflection at the thicknesses 1, 3, 5, 7, &c. and to transmission at the thicknesses 0, 2, 4, 6, 8, &c. for its transmission through the first surface is at the distance 0, and it is transmitted through both together, if their distance be infinitely little, or much less than 1, the disposition to be transmitted at the distances 2, 4, 6, 8, &c. is to be accounted a return of the same disposition which the ray first had at the distance 0, that is, at its transmission through the first refracting surface.

- T This alternate reflection and transmission depends on both the surfaces of very thin plate, because it depends on their distance. For if either surface of a thin plate of Muscovy-glass be wetted, the colours grow faint (285, 0): it must therefore depend upon both.

It is therefore performed at the second surface; u for if it were performed at the first, before the rays arrive at the second, it would not depend on the second.

It is also influenced by some action or disposi- v tion, propagated from the first to the second, because otherwise at the second it would not depend upon the first. And this action or disposition, in its propagation, intermits and returns by equal intervals, because in all its progress it inclines the ray at one distance from the first surface to be reflected by the second, at another to be transmitted by it, and that, by equal intervals, for innumerable vicissitudes.

The returns of the disposition of any ray to be w reflected are termed its fits of easy reflection, and those of its disposition to be transmitted its fits of easy transmission; and the space it passes through between every return, and the next return, the interval of its fits.

Thus, let ODE (fig. 83) represent a transpa- x rent medium, suppose water, upon which the ray AB is incident at a point in the upper surface O , O . Draw the line I , I , and let the interval between it and O , O , be every where equal to the distance between the two surfaces of the plate of water, described in the last chapter (283, F), when the first ring of colour is reflected. Then if the inferior surface of the medium were at I , I , the ray would be reflected upon the same principle as the ring of colour, and therefore at I , I it is in a fit

of easy reflection: Draw the parallel 2, 2 at the same distance from 1, 1, and the distance between 0, 0, and 2, 2 will be that thickness at which in the before-mentioned plate the first ring of colour is transmitted: the ray would therefore be transmitted if the inferior surface were at 2, 2, and consequently it is there in a fit of easy transmission. At 3, 3 it is again in a fit of easy reflection, and by applying the same argument to the equidistant lines 4, 4; 5, 5; 6, 6; 7, 7; 8, 8; it will appear that the ray will be alternately disposed to transmission and reflection; and if the last parallel or the inferior surface be distant from the superior surface 0, 0, by an even number of intervals, the ray will arrive there in a fit of easy transmission and emerge; but if the number be odd, it will arrive in a fit of easy reflection, and return back into the medium. The distance between the lines 0, 0 and 2, 2; 2, 2 and 4, 4, &c. are the intervals of the fits of easy transmission, and the distances between 1, 1 and 3, 3; 3, 3 and 5, 5, &c. are the intervals of the fits of easy reflection.

- r What kind of action or disposition this may be, whether it consist in a circulating or a vibrating motion of the ray or of the medium, or something else, experiments are wanting to determine. But the facts are not the less true on account of our ignorance of the mode of their origin. That truly great man, to whose penetration and industry we are indebted for almost all the knowledge we have of the physical properties of light, has, with
great

great modesty, proposed an hypothesis for the solution of these appearances. It is not without its difficulties, and must therefore be received with the same caution as it was proposed, till experiment shall either confirm it, or substitute another theory in its place.

Sir Isaac Newton's Hypothesis. It may be supposed, that as stones by falling into water put the water into an undulating motion, and all bodies by percussion excite vibrations in the air; so the rays of light, by impinging on any refracting or reflecting surface, excite vibrations in the refracting or reflecting medium or substance, and by exciting them, agitate the solid parts of the refracting or reflecting body, and by agitating them, cause the body to grow warm or hot; that the vibrations thus excited are propagated in the refracting or reflecting medium or substance much after the manner that vibrations are propagated in the air for causing sound, and move faster than the rays, so as to overtake them; and that when any ray is in that part of the vibration which conspires with its motion, it easily breaks through a refracting surface, but when it is in the contrary part of the vibration which impedes its motion, it is easily reflected; and, by consequence, that every ray is successively disposed to be easily reflected or easily transmitted by every vibration which overtakes it.

C H A P. VIII.

OF THE PERMANENT COLOURS OF NATURAL BODIES, AND THE ANALOGY BETWEEN THEM AND THE COLOURS OF THIN TRANSPARENT PLATES.

- A It has already been shewn (272, E), that the colours of natural bodies consist in a disposition to reflect the rays of one sort of light more copiously than the rest. But their constitution, whereby they reflect some rays more copiously than others, remains to be disclosed.
- B Those superficies of transparent bodies reflect the greatest quantity of light, which have the greatest refracting power; that is, which intercede mediums that differ most in their refractive densities. And in the confines of equally refracting mediums there is no reflection.
- C The analogy between reflection and refraction will appear by considering that the most refractive mediums totally reflect the rays of light at less angles of incidence, as was before shewn (270, A). But the truth of the proposition will further appear by observing, that in the common superficies of two transparent mediums, the reflection is stronger or weaker, accordingly as the superficies hath a greater or less refractive power. If any transparent solid be immersed in water, its

its reflection becomes much weaker than before, and still weaker if immersed in a fluid whose refracting power is yet stronger than that of water. If water be distinguished into two parts by an imaginary surface, the reflection in the confine of those two parts is none at all. In the confine of water and ice it is very little; in that of water and oil something greater; in that of water and sal-gemm still greater; and in that of water and glass, or crystal, or other denser substances still greater, accordingly as those mediums differ more or less in their refractive powers. The reason then why uniform pellucid mediums, as water, glass or crystal, have no sensible reflection, but at their external superficies, where they are adjacent to other mediums of a different density, is that all their contiguous parts have one and the same degree of density.

The least parts of almost all natural bodies, are in some measure transparent: and the opacity of bodies arises from the multitude of reflections caused in their internal parts.

This may be easily seen by viewing small substances with the microscope or magnifying glass, for they appear for the most part transparent. And it may also be tried by means of the light received through a hole into a dark chamber. For any substance, how opaque soever, if it be reduced to a sufficient thinness, and applied to the hole, will appear manifestly transparent. Only white metalline bodies must be excepted, which, by reason of

their very great density, seem to reflect almost all the light incident on their first superficies, unless by solution in menstrums, they be reduced into very small particles, and then they also become transparent.

G Between the parts of opaque or coloured bodies are many spaces, either empty or replenished with mediums of other densities; as water between the tinging corpuscles with which any liquor is impregnated, air between the aqueous globules that constitute clouds and mists; and for the most part, spaces void both of air and water, but yet, perhaps, not void of all substance, between the parts of hard bodies.

A The truth of this is evinced by the two precedent propositions (F, G): for, by the second, there are many reflections made by the internal parts of bodies, which would not happen if the parts of those bodies were continued without any such interstices between them; because reflections are only made in superficies which intercede mediums of different densities (293, D).

B A yet farther proof that the opacity of bodies arises from this discontinuation of their parts may be had, by considering that opaque substances become transparent, by filling their pores with any substance of an equal or nearly equal density with their parts. Thus, paper dipped in water or oil, the oculus mundi stone steeped in water, linen cloth oiled or varnished, and many other substances soaked in such liquors as will intimately pervade their

their pores, become by that means more transparent than otherwise; so, on the contrary, the most transparent substances may, by evacuating their pores, or separating their parts, be rendered sufficiently opaque, as salts, or wet paper, or the oculus mundi stone, by being dried; horn, by being scraped; glass, by being reduced to powder, or otherwise flawed; turpentine, by being stirred about with water till they mix imperfectly; and water, by being formed into many small bubbles, either alone in the form of froth, or by shaking it together with oil of turpentine, or some other convenient liquor with which it will not perfectly incorporate.

The parts of bodies and their interstices must not be less than some definite bigness to render them opaque and coloured.

For the opaquest bodies, if their parts be subtilly divided, as metals, by being dissolved in acid menstruums, &c. become perfectly transparent. And it may also be remembered, that the black spot near the point of contact of the two plates of glass being of some considerable breadth, transmitted the whole light where the glasses did not absolutely touch (281, *y*). And the reflection at the thinnest part of the soap-bubble was so insensible as to make that part appear intensely black, by the want of reflected light (284, *i*).

On these grounds it is, that water, salt, glass, & stones, and such like substances, are transparent. For on many considerations they seem to be as full of pores or interstices between their parts as other bodies

are, but yet their parts and interstices to be too small to cause reflection in their common surfaces.

F The transparent parts of bodies, according to their several sizes, must reflect rays of one colour, and transmit those of another, on the same ground that thin plates or bubbles do reflect or transmit those rays. And this appears to be the ground of all their colours.

G For if a thin body or plate, which, being of an even thickness, appears all over of one uniform colour, should be slit into threads, or broken into fragments of the same thickness with the plate; there is no reason why every thread or fragment should not keep its colour, and by consequence, why a heap of those threads or fragments should not constitute a mass or powder of the same colour which the plate exhibited before it was broken. And the parts of all bodies being like so many fragments of a plate, must on the same grounds exhibit the same colours.

H Now, that they do so, will appear by the affinity of their properties. The finely coloured feathers of some birds, and particularly those of peacocks tails, do in the very same part of the feather appear of several colours in several positions of the eye. Likewise the fine-spun webs of some spiders appear coloured; and the fibres of some silks, by varying the position of the eye, do vary their colours. Also the colours of silks, cloths, and other substances which liquids can easily penetrate, become more faint by being wetted, much after the manner of the
plate

plate of Muscovy glass, and recover their vigour again by being dried.

The air reflects the blue rays most plentifully, and must therefore transmit the red, orange, and yellow more copiously than the other rays. If the light of the setting-sun, by passing through a long tract of air, be divested of the more reflexible rays, the green, blue indigo, and violet, the remainder, which is transmitted, will illuminate the western clouds with an orange colour; and as the Sun sets more and more, the tract of air through which the rays must pass becomes longer, the yellow and orange are reflected, and the clouds grow more deeply red, till at length the disappearance of the Sun leaves them of a leaden hue by the reflection of the blue light from the air. A similar change of colour is observed on the snowy tops of the Alps in Switzerland, and the same may be seen, though less strongly, on the eastern and western fronts of white buildings; St. Paul's Church at London is a good object of this kind, and is often at sun-set tinged with a considerable degree of redness. The same cause likewise occasions the Moon in an eclipse to assume a ruddy colour by the light transmitted through the atmosphere (156, N, O).

The parts of bodies, on which their colours depend, are denser than the medium which pervades their interstices.

For if they were not, the variation of colour, arising from the various obliquities of the incident light, (283, E. 285 K) would compound a mixt and imperfect colour, and never so vivid as experience evinces,

evinces. But when the parts are much denser than the ambient medium, this variation is not so considerable; and therefore, the rays which are reflected least obliquely may predominate over the rest, so much as to cause a heap of such particles to appear very intensely of their colour.

M And hence the magnitude of the component parts of natural bodies may be conjectured by their colours.

N For, since the parts of these bodies are of about the same density as water or glass, as by many circumstances is obvious to collect, it is highly probable that they exhibit the same colours with a plate of equal thickness. That colour being known, the thickness may be easily found by the preceding observations.

C H A P. IX.

OF THE INFLECTIONS OF THE RAYS OF LIGHT WHICH PASS IN THE VICINITIES OF BODIES.

O It is observable, that if a beam of the Sun's light be let into a dark room through a very small hole, the shadows of things in this light will be larger than they ought to be if the rays went on by the bodies in straight lines, and that these shadows have three parallel fringes, bands, or ranks of colours adjacent to them. The principal circumstances of the phenomenon are as follow:

P If a beam of the Sun's light be admitted into a darkened chamber through a hole of the breadth
of

of the forty-second part of an inch, or thereabouts, the shadows of hairs, thread, straws, and other small bodies, appear considerably broader than they would be if the light passed by them in strait lines. For example; a hair, whose breadth was the 280th part of an inch, being held in this light at the distance of about twelve feet from the hole, did cast a shadow which, at the distance of four inches from the hair, was the sixtieth part of an inch broad, that is, above four times broader than the hair; and at the distance of ten feet, was the eighth part of an inch broad, that is, thirty-five times broader.

Nor is the effect altered by an alteration in the Q density of the medium contiguous to the hair, for its shadow at like distances was equal, whether it was in the open air, or inclosed between two plates of wet glass, care being had that the incidence and emergence of the ray was perpendicular to the glasses. Scratches on the surface or veins in the body of polished glasses did also cast the like broad shadows. And therefore the great breadth of these shadows must proceed from some other cause than the usual refraction which might arise from any action of the ambient medium.

Let the circle x (fig. 84) represent the middle R of the hair; ADG , BEH , CFI , three rays passing by one side of the hair at several distances; KNQ , LOR , MPS ; three other rays passing by the other side of the hair at the like distances; D , E , F , and N , O , P , the places where the rays are bent in their passage by the hair; G , H , I , and Q , R , S ,
↓
the

the places where the rays fall on a paper, GQ ; rs the breadth of the shadow of the hair cast on the paper; and ri , vs , two rays which fall on the points i and s , without being at all deflected by the action of the hair. Then it is manifest, that all the rays between ri and vs are bent in passing by the hair, and turned aside from the shadow rs , because, if any part of the light were not bent it would fall within the shadow, and there illuminate the paper, contrary to experience. And because, when the paper is at a great distance from the hair, the shadow is broad, and therefore the rays ri and vs are at a great distance from each other, it follows that the hair acts upon the rays of light at a considerable distance in their passing by it. But because the shadow of the hair is much broader in proportion to the distance of the paper from the hair when the paper is nearer to the hair than when it is at a great distance from it, it is evident that the action is stronger on the rays which pass by at least distances, and grows weaker and weaker accordingly as the rays pass by at distances greater and greater, as is represented in the scheme.

- s The shadows of all bodies in this light are bordered with three parallel fringes or bands of coloured light, of which that contiguous to the shadow is broadest and most luminous, and that most remote from it is narrowest, and so faint as scarcely to be visible. If the light be received very obliquely on paper, or any other smooth white body, the colours may be plainly distinguished in this order, viz. the first

first or innermost fringe is violet, and deep blue next the shadow, and then light blue, green and yellow in the middle, and red without. The second fringe is almost contiguous to the first, and the third to the second, and both are blue within, and yellow and red without, but their colours are very faint, especially those of the third. The colours therefore proceed in this order from the shadow, violet, indigo, pale blue, green, yellow, red; blue, yellow, red; pale blue, pale yellow, and red.

If a larger beam of the Sun's light be admitted ^r into a dark chamber, and part of it received on the blade of a sharp knife, whose plane intersects the direction of the beam at right angles, while the other part is suffered to pass by the edge of the knife, and received on a paper at the distance of about three feet; this last light will appear to shoot out or send forth two faint luminous streams both ways into the shadow, somewhat like the tails of comets. These streams being very faint, are so much obscured by the light of the principal direct rays, that it is necessary, in order to see them with any degree of distinctness, to let the direct rays pass through a hole in the paper on to a piece of black cloth. The light of the streams is then perceptible on the paper to the distance of six or eight inches from the Sun's direct light each way, and in all the progress, from that direct light decreases gradually till it becomes insensible.

If two knife-blades, with strait edges, be so ^u fixed or set in a frame, that they may both be
situated

situated in the same plane, their edges parallel, and facing each other, and one of the blades moveable towards or from the other by means of a screw, so that their parallelism may be always preserved, a beam of light may be suffered to pass between their edges, and the appearances are the following: when the knives are at a considerable distance, so that the intromitted beam is broad, the streams of light which shoot both ways into the shadow are scarce visible, for the reason already mentioned, and the edges of the shadows are not bordered with coloured fringes, they becoming so broad that they run into each other, and by joining, form one continued light or whiteness at the beginning of the streams. As the knives approach each other the fringes of colour appear on the confine of each shadow, becoming distincter and larger until they vanish, which happens when the edges are distant somewhat more than the 400th part of an inch. After the fringes have disappeared, the line of light, which was in the middle between them, grows very broad, enlarging itself both ways into the streams of light afore-mentioned; and when the knives are distant above the 400th part of an inch, the light parts in the middle, and leaves a shadow between the two parts. And as the knives still approach each other, the shadow grows broader, and the streams shorter at their inward ends, which are contiguous to the shadow, till upon the contact of the knives

knives the whole light vanishes, leaving its place to the shadow*.

From these and some other experiments of the same tendency, it may be inferred,

That all bodies act upon the particles of light v attracting them when within a certain distance, and at greater distances repelling them; for the two comet-like streams seem to be produced, the one by an attractive power exerted, by which the light is thrown into the shadow of the knife, and the other by a repulsion, by which it is turned towards the contrary part or region.

That these actions are stronger on those rays w which pass nearer the body than on those which pass at greater distances: consequently those rays which were parallel before their arrival in the vicinity of the body being variously deflected, must, after passing, diverge from each other; and, at the limit or distance at which attraction ceases, and repulsion begins, there must be a place at which the passing rays being very little affected by the action of the body, will proceed parallel, as before their arrival in its vicinity.

That this limitation or distance may differ in x rays of different colours, and cause the appearance

* The experiments of Newton on the inflection of light are few, and imperfect. Those who have followed him in this delicate and highly important department of Natural Philosophy, have done little more than add some insulated facts to those observed by him. The law followed by the powers that inflect light, and the limits of its action, are yet unknown.

of

of fringes: for, if the limit be less in the violet rays than in the red rays, the parallel rays of the violet colour will form a fringe, which shall be nearer the shadow of the body than that which is formed by the parallel rays of the red colour: and so of the intermediate colours will be formed intermediate fringes. It must, however, be confessed, that this supposition does not account for the repetition of the same colour at different distances.

C H A P. X.

OF THE POWERS BY WHICH BODIES REFLECT OR REFRACT THE RAYS OF LIGHT.

- Y** THE reflection of light is not caused by its impinging or striking on the solid parts of bodies.
- Z** This will appear by the following considerations. First, That in the passage of light out of glass into air, there is a reflection as strong as in its passage out of air into glass, or rather a little stronger, and by many degrees stronger than in its passage out of glass into water. And it seems not probable, that air should have more reflecting parts than water or glass. But if that should possibly be supposed,
- A** yet it will avail nothing; for the reflection is as strong, or stronger, when the air is drawn away from the glass, as when it is adjacent to it. Secondly, if light in its passage out of glass into air be incident more obliquely (27Q, A) than at an angle of 40 or

41 degrees, it is wholly reflected; if less obliquely, it is in a great measure transmitted. Now, it is not to be imagined that light, at one degree of obliquity, should meet with pores enough in the air to transmit the greater part of it, and at another degree of obliquity, should meet with nothing but parts to reflect it wholly; especially considering that in its passage out of air into glass, how oblique soever be its incidence, it finds pores enough in the glass to transmit the greatest part of it. If any one suppose that it is not reflected by the air, but by the outmost superficial parts of the glass, there is still the same difficulty: besides, that such a supposition is unintelligible, and will also appear to be false, by applying water behind some part of the glass instead of air. For so in a convenient obliquity of the rays, suppose of 45 or 46-degrees, at which they are all reflected where the air is adjacent to the glass, they shall be in great measure transmitted where the water is adjacent to it; which argues that their reflection depends on the constitution of the air and water behind the glass, and not in the striking of the rays upon the parts of the glass. Thirdly, If the colours made by a prism placed at the entrance of a beam of light into a darkened room be successively cast on a second prism (271, c) placed at a distance from the former, in such manner that they are all alike incident upon it, the second prism may be so inclined to the incident rays, that those which are of a blue colour shall be all reflected by it, and yet those of a red colour pretty

copiously transmitted. Now, if the reflection be caused by the parts of air or glass, it may be demanded why, at the same obliquity of incidence, the blue should wholly impinge on those parts, so as to be all reflected, and yet the red find pores enough to be in great measure transmitted. Fourthly, Where two glasses touch one another there is no sensible reflection (281, v), and yet no reason can be given why the rays should not impinge on the parts of the glass as much when contiguous to other glass as when contiguous to air. Fifthly, When the top of a soap water bubble, by the continual subsiding and exhaling of the water, becomes very thin, there is such a little and almost insensible quantity of light reflected from it, that it appears intensely black (284, i); whereas, round about that black spot, where the water is thicker, the reflection is so strong as to make the water seem very white. Nor is it only at the least thickness of thin plates or bubbles, that there is no manifest reflection, but at many other thicknesses continually greater and greater. For we have seen that the rays of the same colour are by turns transmitted at one thickness, and reflected at another thickness for an indeterminate number of successions. And yet, in the superficies of the thin body, where it is of any one thickness, there are as many parts for the rays to impinge on as where it is of any other thickness. Sixthly, If reflection were caused by the parts of reflecting bodies, it would be impossible for thin plates or
bubbles

bubbles at the same place to reflect the rays of one colour, and transmit those of another. For it is not to be imagined, that at one place the rays which, for instance, exhibit a blue colour, should accidentally strike upon the parts, and those which exhibit a red upon the pores, of the body; and then at another place, where the body is either a little thicker or a little thinner, that on the contrary, the blue should hit upon its pores, and the red upon its parts. Lastly, Were the rays of light reflected by im- c
pinging on the solid parts of bodies, their reflections from polished bodies could not be so regular as they are. For in polishing glass with sand, putty, or tripoly, it is not to be imagined that those substances can, by grating and fretting the glass, bring all its least particles to an accurate polish, so that all their surfaces shall be truly plane or truly spherical, and look all the same way, so as together to compose one even surface. The smaller the particles of those substances are, the smaller will be the scratches by which they continually fret and wear away the glass until it be polished; but be they ever so small, they can wear away the glass no otherwise than by grating and scratching it, and breaking the protuberances, and therefore polish it no otherwise than by bringing its roughness to a very fine grain, so that the scratches and frettings of the surface become too small to be visible. And therefore, if light were reflected by impinging upon the solid parts of the glass, it would be scattered as much and as irregularly by the most polished

glass as by the roughest. So that it remains a problem, how glass polished by fretting substances can reflect light so regularly as it does. And this problem is scarce otherwise to be solved than by saying, that the reflection of a ray is effected not by a single point of the reflecting body, but by some power of the body which is evenly diffused all over its surface, and by which it acts upon the ray without immediate contact: for that the parts of bodies do act upon light at a distance, has already been shewn (301, T, U).

D Now, if light be reflected, not by impinging on the solid parts of bodies, but by some other principle, it is probable that as many of its rays as impinge on the solid parts of bodies are not reflected, but stifled or lost in the bodies. For otherwise, we must allow two sorts of reflections. Should all the rays be reflected which impinge on the solid parts of clear water or crystal, those substances would rather have a cloudy colour than a clear transparency. To make bodies look black in all positions, it is necessary that many rays be stopped, retained, and lost in them; and it is difficult to conceive that any rays can be stoppt and stifled in them which do not impinge on their parts.

E Bodies reflect and refract light by one and the same power, variously exercised in various circumstances.

This appears by several considerations. First, Because when light goes out of glass into air as obliquely

obliquely as it can possibly do, if its incidence be made still more oblique, it becomes totally reflected (270, A). For the power of the glass, after it has refracted the light as obliquely as is possible, if the incidence be still made more oblique, becomes too strong to let any of its rays go through, and by consequence causes total reflection. Secondly, Because light is alternately reflected and transmitted by thin plates of glass for many successions (285, L) accordingly as the thickness of the plate increases in an arithmetical progression. For here the thickness of the glass determines whether that power by which glass acts upon light shall cause it to be reflected, or suffer it to be transmitted. And thirdly, Because those surfaces of transparent bodies which have the greatest refracting power do also reflect the greatest quantity of light (292, n, c).

The power by which bodies reflect and refract light, is the same as was shewn to be common to all bodies, and the cause of the inflection of the rays of light passing in their vicinities (303, u). For we must admit no more causes than are true, and sufficient to explain the phenomena (6, 1). Such a cause is this; its existence being proved, and its adequacy to the explanation of the reflection and refraction of light easy to be shewn.

Let cd , (fig. 85) represent the surface of a transparent body A , contiguous either to a vacuum B , or other medium possessing a less power of reflecting or refracting the rays of light. Let EF represent an imaginary surface at such a distance from

CD as to be situated at the limit of attraction, (303, w) that is to say, the space between EF and CD is that in which, if a ray of light pass, it will be attracted by the dense body A , and on the other side towards B , near the line EF , a ray of light will be repelled.

- H Suppose now GH to be a ray of light passing within the rare medium B , obliquely towards the surface CD , and let the line or part KH denote its velocity. This motion may (23, T) be resolved into KI parallel, and IH perpendicular to CD . The attraction or repulsion exerted by the nearest parts of the body A (and the other parts may be neglected) or by those in the surface CD , must be assumed to act in the perpendicular to that surface, because no reason can be given why it should act towards one side more than another. It will therefore alter only the motion IH without affecting KI . When the light approacheth EF it will be repelled; and if the force of repulsion in arriving at EF be greater than would generate the momentum IH , this last motion will be entirely destroyed before the light can arrive at the imaginary surface. The action of the repulsion, after it has destroyed IH , will, whatever may be its law, produce an equal velocity in the opposite direction. Consequently the ray will describe a motion compounded of KI and IL (equal to KI) and in the same direction; that is, it will pass through the line HL , making the angle of reflection IHL equal to the angle of incidence IKK (262, Y).

Again, suppose mn to be a ray of light passing within the rare medium B , which either by the more direct course towards the surface or otherwise has the perpendicular part on of its motion too great to be destroyed by the repulsion experienced in approaching EF . It will pass that imaginary surface, suffering only a diminution of its velocity estimated in the perpendicular on . While it goes forward towards CD , its velocity in the perpendicular will be continually augmented by the attractive force; and if the whole accelerating force exceed the whole retarding force, as in this case experience shews it does, the light will enter and proceed in the dense body with a velocity in the perpendicular qs , greater than it had before in on ; the parallel velocity po or sr still continuing the same. The ray qt will for this reason make a less angle sqr with the perpendicular than before, instead of continuing in the line nu ; that is, it will be refracted towards the perpendicular by entering the dense body (262, A).

Again, suppose vw to be a ray of light passing within the dense body A , obliquely towards the surface CD . Resolve the motion represented by vw into vx and xw , the first parallel, and the latter perpendicular to CD . The ray will pass out of the dense body into the space between EF and CD ; where, if the force of attraction towards CD on a ray during its passage to EF be greater than the momentum xw in the contrary direction, this last motion will be entirely destroyed before the light

can arrive at the imaginary surface. Whence it follows, that for reasons similar to those used in speaking of the ray GH (310, H), the ray vw will be again returned towards cd , with a velocity equal and contrary to xw , which, together with xz , the continuation of the uniform and unaltered velocity yx , will compound the actual motion wz , making the angle of reflection xwz equal to the angle of incidence xwy (262, Y).

- L Lastly, suppose rq to be a ray of light passing within the dense body $^$, which either by the more direct course towards the surface, or otherwise, has the perpendicular part sq of its motion too great to be destroyed by the attraction experienced in its passage to er . It will pass that imaginary surface, suffering only a diminution of its velocity estimated in the perpendicular sq . When it has gone beyond ef , its velocity in the perpendicular will be continually augmented by the repulsive force; and if the whole accelerating force be less than the whole retarding force, as in this case experience shews it is, the light will enter, and proceed in, the rare medium with a velocity in the perpendicular no , less than it had before in sq ; the parallel velocity rs or or still continuing the same. The ray np will for this reason make a greater angle onp with the perpendicular than before; that is, it will be refracted from the perpendicular by entering the rare medium (262, Z, A).

- M From these considerations it is deduced also, that the rays of light are not refracted or reflected

all

all at once, but in refraction bent into a curve by the action of the body, so as to enter the surface of any medium more or less directly than they otherwise would have done, if its density had continued the same through the whole course of the rays. And in reflection, that the force acting in the direction of the perpendicular to the surface of a body, does not destroy the motion of the ray all at once, but bends it back in a curve. Which force, when it has destroyed that part of the motion of the ray which tended perpendicularly towards the common surface of the adjacent mediums, must reflect the ray with an equal angle and degree of velocity on the opposite side of the perpendicular to the point of incidence, or vertex of the curve. This is evident from what has already been said on the composition and resolution of motion (23, 1), and may, perhaps, without entering into particular explanations, be more readily conceived by attending to the motions of bodies projected obliquely from the Earth's surface; for here the ascending or perpendicular part of the motion is gradually destroyed by the continually acting force, and a new, similar, and equal motion is generated in the contrary direction, which, abstracting the effect of the air's resistance, causes the body to fall under an equal angle, and with the same velocity.

If the forces of bodies upon the particles of light be supposed to act equally after the ratio of the masses of the particles, the rays will be all
equally

- equally refracted or reflected, however different
 P their masses, provided their velocities be equal. If the same law of the forces be supposed, and the velocities of the particles be various, those which move with less velocities will suffer a greater deflection than those which move with greater velocities. The varying refrangibility and reflexibility of the rays of light must arise either from the various velocities of the particles themselves, or from the action of bodies on the particles being stronger on some than on others, after the ratio of their
 Q masses. If the various velocities were the cause, the moons of Jupiter, after being eclipsed, ought to appear illuminated with a variety of colours, in succession, as the velocities of their constituent rays caused them respectively to arrive at the eye of the observer: and when light is dispersed, by refraction, into its component colours, the quantity of this dispersion ought in every medium to be equal at equal mean refractions of the whole ray: both which are contrary to experience.
- R Whence it follows, in order to produce the variety of refraction or reflection which happens in the several rays of light, bodies must act on some of the particles of light more strongly than upon others, after the ratio of their masses.

B O O K II.

S E C T. II.

Of Optics.

C H A P. I.

CONCERNING THE REFLECTION AND REFRACTION OF LIGHT BY SURFACES REGULARLY FORMED.

BEFORE the discoveries of Sir Isaac Newton ^A had shewn the composition of white light, the science of optics consisted of propositions in which the rays of light were always supposed to be equally "refrangible or reflexible. And, indeed, though the dispersion of light, when refracted into its component colours, is the greatest obstacle to the perfection of the instruments now made; yet on most occasions, with respect to vision, we may regard a ray of white light as still continuing white, even after refraction. For the colours of the spectrum into which it is dilated, are so near each other; when the incidence is near the perpendicular, that to sense they form a white very little differing from that of the incident ray. But in strict-
ness, B

ness, the general principles of optics are true only of any single kind of rays.

- c** That bodies are visible only by means of the light which they emit or reflect, is too evident to need any particular proof; and that every point of an illuminated surface emits the rays of light in all directions, is clear from the visibility of the surface, to an eye in any position whatsoever: for if any part or sensible point of the surface did not emit light in a supposed or given direction, that point, to an eye placed in that direction, must be invisible. But this effect never happens.
- d** The rays which proceed from a point are necessarily divergent, but if they fall on a reflecting or refracting surface, they will be scattered in such directions as the construction of the surface produces. If the surface be properly formed, the whole beam of rays may proceed, after reflection or refraction, either diverging from some other point, or parallel, or converging to a point.
- f** When the rays which are emitted or proceed from any point are considered, that point is called
- g** the radiant point; when the rays which proceed to any point are considered, that point is called the
- h** focus; and when the rays which proceed from a whole surface or object, are considered, the body of rays which is emitted from any one point, or as much of it as is applied to use, is called a pencil of rays.
- i** Since a pencil of rays emanating from any given point of space, is the means by which the
light

sight assures us, that a body exists at or in that point, it is plain that we are liable to deception in that respect: for if the pencil be so affected, either by reflection or refraction, as to proceed with a different divergency or direction, that is, in the same manner as it would have proceeded if emitted from some other point of space, the sense will refer the place of the object to the point which is in the direction of the last course of the rays (263, c).

Thus, if MR (fig. 86) represent the section of κ a plane mirror, and OB an object, then the pencils OC and BD being reflected at C and D , will proceed to the eye at E , in the same manner as if emitted from points situated at I and M , and the same happening to the pencils which are emitted from the intermediate points between O and B , the sense will refer the place of the object to IM . The same happens by refraction, as is clear from the consideration of fig. 73. (263, E).

If a pencil of rays be rendered convergent, so L as to meet and cross each other in a point, they will afterwards diverge, and the sense will refer the place of the radiant point or object to the focus of the convergent rays, from which the divergence was last made; and that rays of any sort may be rendered thus convergent, either by reflection or refraction, is easily shewn.

Suppose R (fig. 87) to be a point, in any ill- M uminated or luminous object, which emits a pencil consisting of seven rays of light, RA , RB , RC ,

RD ,

$R D, R E, R F, R G$; let the ray $R A$ be received on a speculum, so placed as to reflect it through the point s : let another speculum be adapted to receive and reflect $R B$ also through s ; and, in like manner, let the other rays be reflected through the same point; and the point s will become a radiant point, by means of the divergent rays, and will affect the sense in the same manner as if the rays actually flowed from a body placed there. If the speculums be supposed to touch each other, they will form a polygonal concavity. Suppose now the number of rays, instead of seven, to be infinite; then the adapted reflecting surface $A G$, instead of polygonal, must become curve, by reason of the infinite number of sides. The same reasoning may be applied to rays, which, instead of being emitted from a point, or diverging, fall on the reflecting surface, either converging to a point, or parallel to each other. It is therefore possible to construct a superficies that shall reflect into a focus the rays of light, which, either by converging or diverging, are directed either to or from any particular point.

- N** Upon the same principles may be constructed speculums, which shall cause the rays, after reflection, to diverge from any given point behind the reflecting surface. Those speculums, which cause the rays to become more divergent must be convex, and those which cause them to become more convergent must be concave, as may easily be imagined.

The celebrated Archimedes, at the siege of Syracuse, is said to have destroyed the ships of Marcellus, by a machine composed of speculums. Since a plane speculum, in theory, reflects all the light which is incident upon it, under the same affections with which it was incident; the rays of the Sun, which, as coming from a vastly distant object, may be esteemed parallel, will be reflected parallel to each other; and consequently will heat and illuminate any substance on which they fall after reflection, in the same manner as if the Sun shone directly upon it. Two speculums, which reflect the Sun's light on the same substance, will heat it twice as much as the Sun's direct light. Three will, in like circumstances, heat it three times as much. And, by increasing the number of speculums, a prodigious degree of heat may be produced; more than sufficient to consume and destroy any inflammable substance.

Though a plane speculum in theory is supposed Q to reflect all the light which falls upon it, yet in practice almost half the light is lost, on account of the inaccuracy of the polish, and the want of perfect opacity in the substance of the mirror; on which accounts it happens that a considerable part of the light is scattered in all directions, and another part is absorbed by the body. The R indefatigable Buffon, in the year 1747, was the first of the moderns who constructed a burning machine of this kind. It consisted of 168 quicksilvered glasses or specula, each 8 inches long and 6 broad,

6 broad, so contrived, that the focal distance might be varied, and also the number of glasses, as occasions required. In the month of March, 1747, with 40 glasses he burnt a plank, at the distance of about 70 feet.

s If a body of rays, which either proceeds parallel, or, by converging or diverging, respects a given point, fall on the intercedent surface of two mediums of different refracting powers, the rays may be so refracted, if the surface be rightly formed, as to proceed parallel, or to converge to, or to diverge from, some other point.

T Let the polygonal surface $A B C D E F G$ (fig. 88) represent the surface intercedent between two mediums, the rarer being situated on the side towards R , and the denser towards s ; and let a pencil, composed of seven rays, $R A$, $R B$, $R C$, $R D$, $R E$, $R F$, $R G$, be incident, each ray on a different plane, as represented in the figure. Suppose the ray $R A$ to be received on the surface at A , with an angle of incidence that corresponds to the angle of refraction which deflects the ray to the point s . And suppose the ray $A B$ to be received less obliquely, or at a certain less angle of incidence; its angle of refraction will also be less, and it will proceed to s . And let a similar adjustment of the planes at c , D , &c. be supposed, and the other rays will be refracted to the same point. Or if s be supposed the radiant point, the mediums being as before, v the focus will be at R . It is therefore plain, that rays proceeding out of a rare into a dense medium are

are rendered more convergent by a convex surface, and rays, proceeding out of a dense into a rare medium, are rendered more convergent by a concave surface; and the contrary. Let the pencil consist of an infinite number of rays, and the polygonal surface, adapted to refract it to a point, will, by reason of the infinite number of its sides, become a curve. The same argument may be applied to rays that are either convergent or parallel at their incidence on the refracting surface. Consequently, the intercedent surface of two mediums may be so formed as to refract into a focus, or render parallel, or divergent those rays, which, at their incidence, are either parallel, or do, by converging or diverging, respect any particular point.

From the established laws of reflection and refraction, it is not difficult to investigate the nature of the curves, into which the before-mentioned surfaces ought to be formed. But as the errors w that arise from the use of spherical surfaces are very small, and may be remedied by other means, and the mechanical or practical construction of the required curves is very difficult, the parts of optical instruments are commonly formed spherical.

C H A P. II.

OF DIOPTRICS; OR THE REGULAR REFRACTION
OF LIGHT.

- x** GLASS being a medium denser and more refracting than the air, is used to make the transparent parts of optical instruments which are constructed to act by the principle of refraction. A piece of glass properly figured for that purpose is called a lens, and is distinguished by the nature of its surfaces: thus **A** (fig. 89) is a plano-convex, **B** a double convex, **C** a plano-concave, **D** a double concave, and **E** a convex-concave.
- y** The two first lenses, **A** and **B**, nearly resemble each other in their properties; for they refract converging or parallel rays to a point or focus, and refract diverging rays, so as either to make them meet in a focus or proceed less divergent than before. If **AB** (fig. 71) represent a double convex lens, and **R** a radiant point, then the rays which fall on the lens will be refracted to **F**, if the lens **A** be of the requisite convexity. For the rays that fall on the convex surface **ACD** are rendered more convergent, and are made to converge still more by falling on the concave surface **ADB** (320, **U**). The two following lenses, **C** and **D** (fig. 89), are referred to one species, on account of the resemblance of their properties; for they render the incident rays more divergent than before, and therefore

fore cause diverging or parallel rays to diverge from an imaginary or virtual focus, and refract converging rays, so as either to make them diverge from an imaginary focus, or proceed less convergent than before. If AB (fig. 91) represent a double concave lens, and R a radiant point, then the rays which fall on the lens will be rendered more divergent, and will proceed as if they had proceeded from the point F , which is called the virtual focus. The fifth lens E resembles A and C , if its convexity be deeper, or a portion of a less sphere than its concavity: but if the concavity be deepest, its properties resemble those of C and D .

In the four first lenses, the changes made in the course of the rays are more considerable the more the surfaces are curved; but in the last the changes are more considerable, the more the curvities of the two surfaces differ from each other.

A right line, as RF (fig. 90) passing through the center of any lens, and perpendicular to both its surfaces, is called the axis of the lens. The focus of rays that respect the axis, either by falling parallel to it, or diverging from or converging to a point situated in it, is found in the axis, and is called the principal focus.

A right line drawn from the point of convergence or divergence of any pencil of rays incident on a lens, through the center of the lens, will pass through the focus of that pencil, if the point of

convergence or divergence be not situated far from the axis.

- G The rays of light which diverge from the focus after passing through a lens, will occasion the sense to refer to that point, as if occupied by a lucid object (316, 1); the focus, therefore, may be said to be the picture or image of the radiant point. And as a surface may be conceived to be composed of an indefinite number of radiant points, the like number of focal points will appear, and consequently a surface will be formed that will be
- H the image of the radiant surface. Let $o\ b$ (fig. 92) represent an object, and $l\ n$ a double convex lens; from o and b through c the center, draw the lines $o\ c\ i$ and $b\ c\ m$, and the foci of the points o and b will be found at i and m in those lines (323, F), more or less distant from c , as the curvity of the surfaces of the glass is less or greater. The foci of the radiant points situated between o and b will be found between i and m , by the same process. Consequently an image will be there formed, resembling the object, from each point of which rays of light will diverge in the same manner as from a real object; and its position, by reason that the rays cross at c , will be inverted, or contrary to
- I the object itself, as appears by the figure. And because the triangles $o\ c\ b$ and $i\ c\ m$ are similar, the linear magnitudes of the image and the object be to each other respectively as their distances from the lens; for,

As the side co , or distance of the object from the lens,

Is to the side ob , or length of the object,

. So is the side ci , or distance of the image,

To the side im , or length of the image.

Again, let ob (fig. 93) represent an object, κ and LN a double concave lens; draw oc and bc , and the virtual foci of the points o and b will be found at i and m in those lines (323, F) more or less distant from c , as the curvity of the surfaces of the glass is less or greater. The intermediate points of the object will have their intermediate foci between i and m , and the position of the image will be erect as well as the object. And because L the triangles ocb and icm are similar, the linear magnitudes of the object and image will be as their distances from the lens.

Hence it may be easily conceived, how convex m lenses become burning-glasses. For as the object and image, if viewed from the center of the lens subtend the same angle, and the Sun is seen under an angle of about half a degree, we may readily find the density of the rays which form its image in the focus of any lens. For example, if a lens, n four inches broad, collect the Sun's rays into a focus, at the distance of one foot, or twelve inches, the image will not be more than $\frac{1}{16}$ of an inch broad. The surface of this little circle, therefore, will be 1600 times less than the surface of the lens, and consequently the Sun's light must be so many times denser within that circle. No

wonder, then, that it burns with a degree of violence and ardor far exceeding that of any culinary fire.

- o If a paper or white substance be held in the focus of a convex lens, the several foci of the radiant points of objects situated on the other side of the lens will illuminate as many points on the paper; which illuminated points agreeing in relative situation, intensity, and colour with those of the objects themselves, will depict an exact and lively perspective view of the same, though by reason of the crossing of the rays, it will be inverted. But this phenomenon is scarcely to be seen, if any light be admitted to fall on the paper besides that which passes through the lens; for which purpose the lens may be fixed in the window-shutter of a darkened chamber, as we shall have occasion to remark in future.

C H A P. III.

OF THE EYE; AND OF VISION.

IF the construction of the universe were not so P
evident a proof of the existence of a supremely wise
and benevolent Creator, as to render particular ar-
guments unnecessary, (162, A) the structure of the
eye might be offered as one, by no means of the least.
This instance, among numberless others, demon-
strating that the best performances of art are infi-
nitely short of those which are continually produced
by the divine mechanic.

Though the apparatus, by which the eye is Q
preserved and kept in a state proper for the quick
motion and accurate direction towards the object
to be viewed, is well worth attention and remark;
yet, as it does not immediately come under our
notice as illustrative of the principles of optics, we
shall consider only the globe of the eye, or organ
by which vision is performed.

The eye is composed of several tunics or inte- R
guments, one within the other, and is filled with-
in with transparent humors of different refractive
densities. The external tunic called the sclerotica,
is white on the anterior part, except a circular por-
tion immediately in front, which is transparent,
and more convex than the rest of the eye: this
transparent part is called the cornea. Immediately
adherent to the sclerotica within, is the choroides,

or uvea, which, at the circumference of the cornea, becomes the iris, being expanded over great part of its surface, though not contiguous to it. The iris is composed of two kinds of muscular fibres; the one sort tend like the radii of a circle towards its center, and the others form a number of concentric circles round the same center. The central part of the iris is perforated, and the orifice, which is called the pupil, is of no constant magnitude; for, when a very luminous object is viewed, the circular fibres of the iris contract, and diminish its orifice; and on the other hand, when objects are dark and obscure, the radial fibres of the iris contract, and enlarge the pupil so as to admit a greater quantity of light into the eye. The iris is variously coloured in different persons, but according to no certain rule: in general, they whose hair and complexion are light coloured, have the iris blue or grey; and on the contrary, those whose hair and complexion are dark, have the iris of a deep brown. But what specific difference this may occasion in the sense, or whether any at all, is not discoverable. Within the uvea is another membrane, which at the circumference of the cornea becomes fibrous, and is called the ligamentum ciliare. This ligament is attached to the circumference of a double convex lens, whose axis corresponds with the center of the pupil; and which, by means of the fibres, can be altered in a small degree in position, and perhaps in figure. The lens is termed the crystalline humor; and is included in a very strong and trans-

transparent membrane, called the *arachnoides*. Between the crystalline humor and the cornea is contained a clear transparent fluid, called the *aqueous humor*; and between the crystalline humor and the posterior part or bottom of the eye is included another clear transparent fluid, which is termed the *vitreous humor*. The refractive density of the crystalline is greater than those of the humors that surround it. On the side next to the nose a nerve is inserted in the bottom of each eye, about twenty-five degrees from the axis of the crystalline, which, after entering the eye, is spread into an exceeding fine coat of network, termed the *retina*. Lastly; a very black mucus or slime is spread over all the internal parts of the eye, that are not transparent, except the anterior part of the *iris*, which, as before observed, is coloured.

In the figure, the three concentric circles *A B C* s (fig. 94) represent the coats of the eye. The external coat, or *sclerotica*, is transparent, and more convex between *A* and *B*, *A K B* being the cornea. The second tunic, or *uvea* is fibrous between *D* and *I*, and between *G* and *H*, and is there called the *iris*; the hole *I H* is the pupil. The third coat becomes fibrous between *D* and *E*, and between *G* and *F*, being there called the *ligamentum ciliare*; and is attached to the circumference of the lens or crystalline humour *E F*. The cavity or chamber *A E F B* is filled with the aqueous humor, and the chamber *D N G F E* is filled with the vitreous humor. At *N* is inserted the optic nerve, the expansion of which,

which, over the internal surface DNO , is the retina.

T The manner in which the eye acts upon the rays of light may be thus explained. Let OL represent an object, and suppose a pencil of light to proceed from O , and enter the eye; then, because the cornea is a convex concave lens, whose convexity is greatest, (323, c) the rays will be rendered more convergent in passing through it; and if the crystalline be properly formed, they will be refracted by it into a focus at c on the retina. The same will happen to the pencil which proceeds from L , whose focus will be m ; and the foci of the intermediate points will be between m and c : consequently an inverted picture or image will be formed on the retina, and sensation be produced by the action of the light on the expansion of the optic nerve, which from thence is conveyed to the

U sensorium. And that the parts of the eye are adapted to produce such an image, appears likewise from experiment: for if the tunica sclerotica be carefully taken away from the back of the eye of any animal, the inverted picture of external objects may be seen on the thin membranes which remain. Neither is the inversion of the image any obstacle to the mind's conceiving that the object is erect; for a focus at m may be considered as the indication of the existence of a radiant point at L , and a focus at c may indicate the existence of a radiant point at O ; and so of others, the mind contem-

V plating the object itself, and not the image; besides which,

which, we have notions respecting position that are not derived from the sight, whence we judge whether a wall is perpendicular or a plane level, &c. These notions are derived from a perception of the direction in which gravity constantly acts; to which direction we always refer. Whence it happens, that though the position of the eye be ever so much changed, the idea of the position of objects in view remains unaltered. For example; if an observer view an upright pole or staff, the image of the pole on the retina will be in a line at right angles to the opening of the eyelids, provided he holds his head upright; but if he vary the position of his head, the image will be formed in a different position, and upon a different part of the retina: notwithstanding which, he constantly imagines the pole to be erect and unaltered.

Because the foci of rays that differ in divergence x are found at different distances from the lens, those which diverge less coming to a focus sooner than those which diverge more, it is necessary that the eye should be adapted so as to act upon the rays that arrive from points at various distances, and to bring them to a focus upon the retina. The natural structure of the eye is such, that parallel rays have their focus on the retina; and when the proximity of any object causes its rays to fall with a greater divergency, the pupil of the eye contracts and excludes the most divergent rays, at the same time that the crystalline is brought forward, and perhaps rendered more convex by means of the
ligamentum

- ligamentum ciliare, by which provisions the focus
 y still falls on the retina. This adjustment of the eye to the distances of objects gives the reason why we cannot view a near and a distant object at the same time; for, if a hair be held at a few inches distance between the eye and a remote object, suppose a tree at half a mile distance, the tree will appear confused and indistinct when the attention is fixed on the hair, and the same will be the case with the hair when the attention is fixed on the distant tree.
- z There are some eyes naturally so defective, that they cannot effect this adjustment. Those which are replete with humors have the cornea and crystalline too convex, so that the pencils come to their foci before their arrival at the retina, where they fall in small circular spaces instead of points, and by their interference render the image confused: on the other hand, if the humors be deficient in quantity, the cornea and crystalline are too flat, and the pencils of rays not being sufficiently refracted, arrive at the retina before their union in their foci; whence arises the same confusion in the image as in the former case. They whose eyes are imperfect in the first manner are called myopes, from their winking or closing their eyelids, but more commonly near-sighted, because they see very near objects distinctly, the divergency of the rays causing their foci to fall on the retina. They whose eyes are too flat are called presbytæ, because the imperfection of the sight of old men
 being

being occasioned by a decay of the humors, is generally of this kind. Both these imperfections may in a great measure be remedied by the use of proper spectacles. Since the rays converge too soon in the eyes of myopes, it is plain that a concave lens interposed between the object and the eye will cause the rays to fall more divergent, and consequently will prevent their converging to a focus before their arrival at the retina. And the rays may be made to converge sooner in the eyes of presbytæ, by means of convex spectacles, so that they, being already convergent when they enter the eye, will be sufficiently refracted by the cornea and crystalline to have their focus on the retina, and cause distinct vision.

These imperfections are much more frequently the consequences of habit than is generally imagined. Studious men are generally near-sighted, whereas sailors, sportsmen, and others, who are used to fix their attention on remote objects, are more subject to the contrary defect. The eyes of old men have another defect, namely, rigidity, or a want of the power of adjustment, so that it often happens that they require concaves for distant and convex lenses for near objects, being capable only of seeing objects distinctly with the naked eye that are at a moderate distance. Every one should avoid the use of spectacles as much as possible. For, though they render vision more distinct, yet, they never fail to increase the defect of the eye, so as in time

to render it almost impossible to see without them with any degree of distinctness.

- B The eyes of various animals are accommodated with great skill to the exigencies of their situations. In fishes the cornea is almost flat, that it may be no obstacle to their speed in the water, but this is compensated by the crystalline, which is spherical, and therefore adapted to perform the whole necessary refraction of the rays. And in cats and some other animals that prey in the dark, the pupil of the eye is so variable as to admit more than an hundred times the quantity of light at one time than another. The human eye admits more than ten times the quantity of light at one time than at another, and perhaps the differences may be much greater in very dark places: it is not improbable but that the iris may be then almost entirely drawn back, and the pupil expanded to the whole surface of the cornea.

C H A P. IV.

OF REFRACTING MICROSCOPES; OR THE DIOPTRIC
INSTRUMENTS, BY MEANS OF WHICH SMALL
AND NEAR OBJECTS ARE MAGNIFIED.

THE apparent magnitude of any object is mea- c
sured by the angle under which it is viewed by
the eye; consequently the apparent magnitudes of
two or more objects may be the same, or may
differ in any proportion, let their real magnitudes
be what they will. Thus, the apparent magnitudes
of CD , FG , and HI (fig. 95) are equal when view-
ed by the eye at E , because they are seen under the
same angle, though their real magnitudes are very
different: and the apparent magnitude of AB is
greater than those of the former three, because it
subtends a greater angle, though its real magnitude
is equal to that of CD , and less than those of FG
and HI .

The image of any object on the retina will be D
greater or less in proportion to its apparent mag-
nitude, and therefore the same object is seen more
enlarged and distinct the nearer it is brought to
the eye, provided its distance be sufficiently great
for the rays to fall nearly parallel on the pupil: at
less distances it continues to be enlarged, but is
confused. The least distance is about six inches.
The eye can just distinguish objects that subtend
an angle of half a minute of a degree, in which

case the image on the retina is less than the $\frac{1}{120}$ part of an inch broad, and the object, supposing it six inches distant, about the $\frac{1}{120}$ part of an inch broad. And all smaller objects are invisible to the naked eye.

- 2 The instruments by which those smaller objects are rendered visible are called microscopes, and are constructed in two different methods. The one is, by the interposition of a convex lens, between the object and the eye, to render it distinct at a less distance than six inches, by which means its apparent magnitude increases as the distance is diminished: and the other is, by placing the object so with respect to a convex lens that its focal image may be much greater than itself, and contemplating that image instead of the object. The first are called simple or single microscopes, and the latter compound or double.

Let EY (fig. 96) represent the eye, and OB a small object situated very near, so that the angle of its apparent magnitude OCB may be large. Then its image on the retina IM will also be large; but because the pencils of rays are too divergent to be collected into their foci on the retina, it will be very confused and indistinct. Let the convex lens AS (fig. 97) be interposed, so that the distance between it and the object may be equal to the focal length at which parallel rays would unite, and the rays which diverge from the object and pass through the lens will afterwards proceed, and consequently enter the eye, parallel: they will therefore unite, and

and form a distinct image on the retina, and the object will be clearly seen, though if removed to the distance of six inches, its smallness would render it invisible. And since the apparent magnitudes of objects that subtend small angles are nearly in the inverse proportion of their distances, if the real magnitudes be equal, the proportion in which the object is magnified will be as six inches to its distance from the eye. Whence it follows, that the most convex lenses, having the shortest focal distance of parallel rays, must magnify the most; for they permit the object to approach nearer the eye than those do which are flatter. When the lens is not held close to the eye, the object is amplified somewhat more; because the pencils, which pass at a distance from the center of the lens, are refracted inwards toward the axis, and consequently seem to come from points more remote from the center of the object, as may be seen in fig. 98, where the pencils which are emitted from o and b, are refracted inwards, and seem to come from the points i and m.

A drop of water is a microscope of this kind, by reason of its convex surface; for, if a small hole be made in a plate of metal, or other thin substance, and carefully filled with a drop of water, small objects may be seen through it very distinct, and much magnified. But there are some difficulties in the management of these, which small glasses are free from, and therefore they are not much used. In fact, cheapness is their principal recommendation.

- K The compound microscope, by means of which the image is contemplated instead of the object, is of two kinds, the solar and the common double
- L microscope. The solar microscope is thus constructed: let AC (fig. 99) represent the side of a darkened chamber, LN a convex lens, fixed opposite a perforation in AC , OB a small object, and PQ a white screen placed within the chamber opposite to the lens; then, if the object be placed at a due distance from the lens, the pencil of light which proceeds from the point O will converge to a focus on the screen at I , and the pencil which proceeds from the point B will converge to a focus at M , and the intermediate points of the object will be depicted between I and M , forming a picture which will be as much larger than the object in proportion as the distance of the screen exceeds that of the image from the lens (324, 1). This is the principle on which the instrument acts, but it is usual to add other auxiliary parts as a lens or speculum to illuminate the object by converging the Sun's light upon it, &c. which cannot, with sufficient brevity, be here enlarged upon. The solar microscope is by far the most pleasing in its effects, and least offensive to the eyes of any in use.
- M In the common double microscope the image is contemplated instead of the object, being viewed through a single lens in the same manner as the object in a single microscope. Thus,
- N Let LN (fig. 100) represent a double convex lens, and OB a small object, so applied, that the pencils

pencils of rays which emerge from it, and pass through the lens, may converge to their respective foci, and form an inverted image at $1M$. This image will be as much larger than the object in proportion as its distance exceeds that of the object from the lens ($324, 1$), and, if it be viewed through the lens FG , will again be magnified upon the principle of the single microscope ($336, F$) in proportion as its distance from the eye is less than six inches; the image formed by the first lens, which is called the object-glass, serving instead of an object for the second, or eye-glass. But it is to be noted, that the image formed in the focus of a lens differs from the real object in a very essential particular; that is to say, the light being emitted from the object in every direction, renders it visible to an eye placed in any position, but the points of the image formed by a lens or mirror emitting no more than a small conical body of rays, which arrives from the glass, can be visible only when the eye is situated within its confine. Thus, the pencil which is emitted from B in the object, and is made to converge by the lens to M , proceeds afterwards diverging towards H , and therefore never arrives at the lens FG , nor enters the eye at E . But the pencils that proceed from the points O and b will be received on the lens FG , and by it carried, parallel, to the eye; consequently the correspondent points of the image i and m will be visible, and those which are situate farther out towards I and M

Q will not be seen. This quantity of the image IM , or visible area, is called the field of view.

R Hence it appears, that if the image IM be large, a very small part of it will be visible, because the pencils of rays will for the most part fall without the eye-glass FG . And it is likewise plain, that a remedy which would cause the pencils, that proceed from the extremes O and B of the object, to arrive at the eye will render a greater part of it visible; or, in other words, enlarge the field of view.

S This is effected by the interposition of a broad lens DE (fig. 101) of a proper curvature at a small distance from the focal image. For, by that means the pencil BM , which would otherwise have proceeded towards H , is refracted to the eye, as delineated in the figure, and the mind conceives from thence existence of a radiant point at P , from which the rays last proceeded (316, 1). In the same manner, the other extreme of the image is seen at Q , and the intermediate points are also rendered
T visible. On these considerations it is, that compound portable microscopes are usually made to consist of an object lens, LN , by which the image is formed, enlarged, and inverted, an amplifying lens, DE , by which the field of view is enlarged, and an eye-glass or lens, by means of which the eye is allowed to approach very near, and consequently to view the image under a very great angle of apparent magnitude*.

* The aberration of the refracted rays from the true focus, which arises from the spherical figure of the lens, DE , the prismatic

The magic lantern is a microscope upon the same principles as the solar microscope, and may be used with good effect for magnifying small transparent objects; but in general it is applied to the purpose of amusement, by casting the species or image of a small transparent painting on glass upon a white wall or screen, at the focal distance from the instrument. After what has already been said, it will be easy to understand the following description of its component parts.

In the inside of a box or lantern is placed the candle or lamp *c* (fig. 102) whose light passes through the plano-convex lens *NN*, and strongly illuminates the object *OB*, which is a transparent painting on glass, inverted and moveable before *NN*, by means of a sliding piece in which the glass is set or fixed. This illumination is still more increased by the reflection of light from a concave mirror, *ss*, placed at the other end of the box, that causes the light to fall upon the lens *NN*, as represented in the figure. Lastly, a lens *LL*, fixed in a sliding tube, is brought to the requisite distance from the object *OB*, and a large erect image *IM* is formed upon the opposite wall.

matic colours that are separated very much, and the loss of light by reflection, which is most considerable when the refraction is greatest, are the causes why in the best double microscopes three or more lenses are substituted instead of the single amplifying lens, &c.

C H A P. V.

OF REFRACTING TELESCOPES; OR THE DIOPTRIC INSTRUMENTS, BY MEANS OF WHICH REMOTE OBJECTS ARE RENDERED LARGE AND DISTINCT TO THE VIEW.

- w As the microscope is calculated to obviate the defects of vision with regard to objects, whose angles of apparent magnitude are too small for sight on account of the smallness of the objects themselves, so telescopes are adapted to improve the sense with respect to objects, whose angles of apparent magnitude are too small for sight by reason of their remoteness or distance. The intention of both instruments is the same, namely, to increase that angle, and, by consequence, the telescope differs very little from the compound microscope, except in some particulars of convenience.
- x Let LN (fig. 103) represent a convex lens, and OB a distant object; then the pencils of rays will be collected into their respective foci, and form the inverted image IM , to which the eye, by means of the lens EF , may approach so near as to view it very large and distinct. This is the common astronomical telescope.
- y But, as it is inconvenient to view objects on the earth inverted, there are usually contrivances annexed to the telescope by which the image becomes erect as well as the object. The simplest of these
- is

is the following, where a concave is substituted instead of the convex eye-glass.

Let LN (fig. 104) represent the object-glass z as before, and OB a distant object. Then the pencils from the respective points of the object would converge to their foci, and form the inverted image IM , if the lens EE were not interposed. But the lens EE being a double concave, occasions the rays to diverge more than before; so that the rays which are emitted from B in the object, instead of converging to M , are made to proceed parallel towards H . For the same reason the rays from O are made to proceed parallel towards K ; the intermediate pencils being affected in the same manner. Now, since parallel rays cause distinct vision, it is plain, that an eye placed in the pencil H , will conceive it to be emitted from some point, suppose m , situated in the last direction of the rays, and the image of B will be seen at m . By the same argument, the image of O will be seen at i , by an eye situated at K , and the like for the intermediate points. Therefore, an image will be seen at im , erect or similarly situated with the object itself.

This telescope represents objects very bright and clear, and as much magnified as the other does, but is unpleasant in its use, on account of the contracted field of view. For the pencils, being rendered divergent with respect to each other, pass mostly on one or the other side, without entering the pupil of the eye, and therefore a very small

part of the image can be seen at once: thus, if the eye be at n , it will view the point m , and if it be moved towards k , it will see in succession all the parts of the image towards i : but, as the pupil of the eye is not broad enough to receive the pencils n and k at the same time, the points m and i cannot be seen at once. The larger the pupil and the nearer it is placed to the eye-glass, the more pencils enter the eye at once. Consequently the field of view is largest under these circumstances, and in all other cases less.

- B By the addition of two eye-glasses to the astronomical telescope, it is adapted to terrestrial objects, the field of view remaining the same. Thus, the lens FF (fig. 105) which is similar to EE , being placed at twice the focal distance for parallel rays from EE , receives the pencils of parallel rays after they have crossed each other at x , and forms an image at im , similar and equal to IM , but contrary in position, or erect; which last image is viewed by the lens GG . This is the common telescope, and though, by reason of the number of lenses, it does not represent objects so bright as the foregoing, yet, its ample field of view makes it much more pleasing and useful*.

* The eye piece of telescopes is usually fitted up with five or more lenses, for reasons similar to those mentioned in the note, on page 340. Their distances are often adjusted in such a manner that they magnify the first image IM on the compound microscope principle.

The

The exhalations which continually arise from the Earth, render the air less transparent, (293) especially near the Earth, where the mixture is less complete, and therefore the celestial bodies are seen much more obscure when in the horizon than when at any considerable elevation; for in the first case, they are viewed through that part of the atmosphere which is contiguous to the surface of the Earth, and in the latter through a part which is at a greater distance. But this obscurity is the least part of the inconvenience. The rising exhalations have a kind of undulating motion, like that of smoke or steam, so that objects seen through them appear to have a tremulous or dancing motion, which is sensible even to the naked eye, if distant objects be viewed in a hot summer's day. Hence also the stars twinkle, and the shadow of lofty buildings have a tremulous motion. In telescopes' this effect is still more perceptible, inasmuch as to render them intirely usefess, for terrestrial objects, when they augment the apparent magnitude more than eighty times*. But when objects in the heavens are viewed at any considerable altitude, instruments may be used which magnify many thousands of times.

* That accurate and enlightened astronomer, Alexander Aubert, Esq. observes, that this undulation is the greatest when the telescope is not placed in the open air, but within a room. For the temperature of the room being seldom correspondent with that which obtains abroad, there is almost always a considerable undulation produced at the window where the streams of hot and cold air mix. Herschel uses his telescopes in the open air.

From

F From this want of transparency in the atmosphere arises that gradual diminution in the light of objects, which painters call the aerial perspective; for, if the air were perfectly transparent, an object would be equally luminous at all distances, because the visible area and the density of light decrease in the same proportion, namely, as the square

G of the distance. It is from this gradual diminution of light, together with the angle of apparent magnitude, that we estimate distances; and because the celestial bodies, when near the horizon, are more obscure, for the reason urged above (345, F), though their respective apparent magnitudes remain unaltered, or in a small degree diminished, we adopt the notion of their being actually larger at

H that time. Thus, likewise, men seen through a mist appear gigantic, the obscurity causing us to imagine them more distant than they really are.

I But, in the case of the heavenly bodies, there is another circumstance that tends to deceive us in our judgment of the distance: we conceive the sky to be a concave dome; and as the clouds towards the horizon are evidently more distant than those near the zenith, we imagine the horizontal radius

K to be much longer than the vertical. From this notion we regulate our ideas with regard to the distance of the heavenly bodies, at different altitudes, which distance, we suppose to be greater than they are near the horizon, and we are consequently led to imagine, that they are larger at that time.

By

By the solar microscope and magic lanthorn, L we have seen that the species of near objects may be cast on a screen in a darkened chamber. The camera obscura has the same relation to the telescope, as the solar microscope has to the common double microscope, and is thus constructed.

Let CD (fig. 106) represent a darkened chamber perforated at L , where a convex lens is fixed, the curvity of which is such, that the focus of parallel rays fall upon the opposite wall. Then, if AB be an object at such a distance, that the rays which proceed from any given point of its surface to the lens L , may be esteemed parallel, an inverted picture will be formed on the opposite wall. For the pencil which proceeds from A will converge to a , and the pencil which proceeds from B , will converge to b , and the intermediate points of the object will be depicted between a and b . This is one of the most pleasing and delightful experiments in optics, and never fails to strike the beholder with surprise and admiration. Its only defect is the inverted position of the picture, which may be remedied by several methods. But as they all tend to make the image less lively, they are seldom used.

C H A P. VI.

OF THE IMPERFECTIONS OF TELESCOPES, AND
THEIR REMEDIES; AND OF THE ACHROMATIC
TELESCOPE.

N SINCE the construction of a telescope consists in nothing more than viewing, by means of a microscope or eye-glass, the image which is formed in the focus of the object-glass; it may seem easy to make a telescope with a given object-glass, that shall magnify in any assignable degree. For, if the eye-glass be rendered more and more convex, the eye may be permitted to approach nearer and nearer to the image, and consequently to view it under an angle of apparent magnitude that shall be greater and greater, as required. But this is
 o unattainable on two several accounts. The first is, that spherical surfaces do not refract the rays of light accurately to a point, as has already been observed; and the second and most consequential is, that the rays of compounded light, being differently refrangible, come to their respective foci at different distances from the glass, the more refrangible rays converging sooner than those which are less refrangible. This is evident from what has already been said on that subject,
 q and is likewise confirmed by experiment; for a paper, painted intensely red, and properly illuminated,

nated, will cast its species, by means of a lens, on a screen at a greater distance than will another blue paper by the same lens in like circumstances. And here it may be noted, that the lens proper for this experiment must be very flat, or a portion of the surface of a large sphere. Hence the species or image of a white object may be said to consist of an indefinite number of coloured images, the violet being nearest, and the red farthest from the lens, and the images of intermediate colours at intermediate distances. The aggregate, or image itself, must therefore be in some degree confused, and this confusion, being very much increased by the magnifying power, or eye-glass, renders it necessary to use an eye-glass of a certain limited convexity to a given object-glass. For which reason, if it be required to construct a telescope that shall magnify objects in a greater degree than a given telescope, the object-glass must be less convex, and of consequence its focal distance longer. Thus an object-glass of 4 feet focal length will bear an eye-glass of about $1\frac{1}{4}$ inch focus, and will magnify objects in length or diameter 40 times: one of 25 feet focal length will bear an eye-glass of 3 inches focus, and magnifies 100 times; and one of 100 feet will bear an eye-glass of six inches, and magnifies 200 times. It is also necessary to limit the aperture of the object-glass, to exclude those rays which are incident at too great distances from the center; for those, being more refracted, are more particularly subject

ject to the irregularities which arise, either from the figure of the glass or the unequal refraction of light. The diameter of the apertures of object lenses, of equal goodness, should be as the square roots of their focal lengths.

- v The great inconvenience and difficulty of managing the longer telescopes, occasioned the philosophic world to fix their thoughts upon the means of converging the rays of light without separating them into their component colours. The expedients for that purpose were first perfected by
- w Sir Isaac Newton and Mr. Dollond. The focal image in the telescope of Sir Isaac Newton is formed by reflection from speculums or mirrors, and being therefore free from the irregular convergency of the various rays of light, will admit of a much larger aperture, and bear the application of a very
- x great magnifying power. The difficulties which attend this instrument, are the tarnishing of the metalline speculums, and the very great accuracy required in giving them the true figure, for an error in a reflecting surface affects the direction of the rays much more than a like error in a re-
- y fracting surface. Yet this telescope is, notwithstanding,
- z the best in use. Mr. Dollond's invention consists in the use of a compound object-glass, which is usually termed achromatic, or colourless, from its property; and the principal imperfection in the practice, is the difficulty of procuring glass that shall be uniformly of the same refractive density. As we are now speaking of
- dioptrics,

dioptrics, it will be more regular to describe the achromatic telescope first, and refer the other to its place, where we shall explain the properties of instruments that act on the principle of reflection.

Because the component rays of light differ ^A from each other in refrangibility, they are separated from each other by refraction, and because they are all refracted so as to preserve a constant ratio between the sines of the angles of incidence and refraction, that separation must be greatest when the whole beam of light is most deflected from its course. From hence opticians have concluded, and there is a passage in Sir Isaac Newton's* optics, that seems to confirm the opinion, that prisms, which deflect the whole beam of light equally out of its course at like incidences, will, however different their refractive densities, occasion also an equal separation or divergency of the component rays: or in other words, that if the emergent refracted light from the surface of a given prism be immediately received on the surface of a second prism, which shall refract it equally in the contrary direction, so that at its emergence, it shall proceed parallel to the first incident beam, this last emergent light will continue white, however different the matter of the second prism may be from that of the first. But this Mr. Dol- ^B lond has shewn to be ill-founded, for, by his experiments it appears, that the different kinds of

* Book I. Part 2. Experiment VIII.

glass differ extremely with respect to the divergency of colours produced by equal refractions. He found that two prisms, one of white flint-glass, whose refracting angle was about 25 degrees, and another of crown-glass, whose refracting angle was about 29 degrees, refracted the beam of light nearly alike, but that the divergency of colour in the white-flint was considerably more than in the crown-glass; so that, when they were applied together, to refract contrary ways, and a beam of light transmitted through them, though the emergent continued parallel to the incident part, it was, notwithstanding, separated into component colours. Whence he inferred, that, in order to render the emergent beam white, it is necessary that the refracting angle of the prism of crown-glass should be increased; and by repeated experiments, he discovered the exact quantity. But this colourless emergent light was not then, by reason of the increased angle of the prism of crown-glass, parallel to the incident ray, but was refracted towards the base of the last mentioned prism.

- c By these means he obtained a theory, in which refraction was performed without any separation or divergency of colour, and which it was not difficult to apply in the construction of the object-
 D glasses of telescopes. Let $A B E D$ (fig. 107) represent a double concave lens of white flint-glass, and $A G D F$ a double concave of crown-glass; then the parts of the lenses which are on the same side of the common axis, namely, $A C B$ and $A F G$, may be

be conceived to act like two prisms, which refract contrary ways, and if the excess of refraction in the crown-glass AFG be such as precisely to destroy the divergency of colour caused by the flint-glass ACB , the incident ray SH will be refracted to x , without any production of colour. The same is also true of the ray sh , and of all the other incident rays, and consequently the whole focal image formed by this compound object-glass will be achromatic, or free from colour which might arise from refraction. It will therefore bear a larger aperture, and greater magnifying power, and of course enlarge objects much more than a common refracting telescope of the same length.

It is more convenient on several accounts to combine three lenses together, one double concave of flint-glass between two convexes of different kinds of crown-glass; and the glasses may be so adjusted to each other, as not only to form a focal image without the prismatic colours, but also free from the defects which in other lenses arise from their spherical figure.

The greatest impediment to the construction of large achromatic telescopes, as has been observed, (350, 2) is the want of a flint-glass of an uniform refracting density. Fortunately for Dollond, this kind of glass was procurable, when he began to make achromatic telescopes, though the attempts

of many ingenious chemists have since been exerted to make it, without much success*.

C H A P. VII.

OF CATOPTRICS, OR THE REGULAR REFLECTION OF LIGHT; AND OF THE REFLECTING TELESCOPE.

e It has been shewn, (317, M) that a surface may be constructed that shall reflect the rays of a given pencil of light, so as to make them either converge to a point, diverge from a point, or proceed parallel to each other. This surface may be either plane or curved.

11 A plane mirror reflects a pencil of light under the same circumstances as it was incident; that is to say, if a pencil, which emanates from a given point, be incident on the mirror, it is reflected so, that its rays proceed with the same divergency from another point, whose distance behind the mirror is equal to the distance of the radiant point

* The author has been informed, that the glass employed by Dollond in the fabrication of his best telescopes, was all of the same melting, or made at one time; and that, excepting this particular treasure, casually obtained, good dense glass for achromatic purposes was always as difficult to be procured as it is now.

before the mirror from the place of incidence: and if the pencils of rays, which emanate from a given surface, be incident on the mirror, they will be reflected so as to preserve the same inclinations to each other as before, and therefore will appear, to proceed from a surface, whose magnitude and distance behind the mirror are exactly equal to those of the radiant surface (317, κ). Hence it is, that plane mirrors reflect the species of objects, which are equal, like, and similar in position with the objects themselves.

Concave mirrors render the pencils of rays, \mathbf{r} which are incident upon them, more convergent, and convex mirrors render them more divergent, (318, ϕ). If the mirrors be regularly formed, according to the proper curve, the convergent or divergent light of any pencil after reflection will respect some particular point of space.

A portion of a sphere, whose breadth is about κ fifteen degrees, differs very little from the curve surface, by which parallel rays would be made by reflection to converge to, or diverge from, a point. It is therefore in many cases used for that purpose, as being much easier to construct.

There is a great resemblance between the properties of convex lenses and concave mirrors, and between the properties of concave lenses and convex mirrors. Convex lenses and concave mirrors form an inverted focal image of any remote object by the convergence of the pencils of rays: concave lenses and convex mirrors do, in general, form an

erect image in the virtual focus, by the divergence of the pencils of rays. In those instruments, whose performances are the effects of reflection, the concave mirror is substituted in the place of the convex lens, and the convex mirror may be used instead of the concave lens: but their dispositions with respect to each other, when combined, must necessarily differ from those of lenses, on account of the opacity of the one, and the transparency of the other.

Q Let A R (fig. 108) represent the polished spherical surface of a concave mirror, and o B an object situated without the center of the mirror; then the pencil of rays, which is emitted from the point o, will fall on the mirror; and after reflection, converge to the focus I; the pencil from B will converge to M, and the like will happen to those emitted from the intermediate points, whose foci will be found between I and M. There will consequently be formed before the mirror an inverted focal image, resembling that which is formed by a convex lens (324, H).

R Let A R (fig. 109) represent the polished spherical surface of a convex mirror, and o B an object: then the pencil of rays, which is emitted from the point o, will fall on the mirror, and after reflection diverge from the virtual focus I; the pencil from B will emerge from M, and the like will happen to those emitted from the intermediate points, whose virtual foci will be found between I and M. There will consequently be
formed

formed behind the mirror an erect focal image, resembling that which is formed by a concave lens.

Let AR (fig. 110) represent a concave mirror, s whose center is c , and OB an object situated without the center. Through the center c , from o , draw the line ON , which will be perpendicular to the mirror at N , and will therefore represent both the incident and reflected ray, which proceeds from o , and is reflected at N : the focal representation or image of o will consequently be found in that line. Through c , from B , draw the line BV , and by the same reasoning the focal image of B will be found in that line. Draw the line or ray OV , and it will be reflected so as to cross the ray ON at I , the angle of reflection IVC being equal to the angle of incidence $OV C$. This intersection of the rays determines the focal point of o , which is I . From B to N draw the ray BN , and its reflection will determine the focus of B , which is M , and the image will be inverted.

Let AR (fig. 111) represent a convex mirror, T and the other representations and construction be as in the last figure. The focal representations of o and B will be found in the lines oc and BC , and the reflected part of the ray OV will virtually cross the line oc at I ; the reflected part of the ray BN will also virtually cross the line BC at M . These intersections will determine the place of the focal image IM , which will be erect.

u Hence it appears to be the property of these mirrors, that the object and the image, if viewed from the center of the sphere, are seen under equal angles; for, the angle $o c b$ is equal to the angle $i c m$; and that the object and image, if viewed from the point of reflection, are seen under equal angles; for, the angle $o v b$ is equal to the angle $i v m$. From this it is easy to find the position and magnitude of the focal image, if the position and magnitude of the object, and the diameter of the sphere, of which the mirror is a part, be known.

w The reflecting telescope, which was made by Sir Isaac Newton, was of the following form.

Let $DEFG$ (fig. 112) represent a tube; at one end of which is placed the concave mirror ΔR , and let $o b$ represent a distant object; then the pencils, which are emitted from the several points of its surface, will be collected, and form an inverted image $i m$. But by the interposition of the plane mirror κc , the rays are reflected, and the image is formed at $i m$, which is seen very much magnified by means of the plano-convex lens at L .

x The immensely powerful telescopes of Herschel are on this construction. This capital artist, and most assiduous astronomer, has made several speculums, which are so perfect as to bear a magnifying power of more than six thousand times in diameter on some of the fixed stars*. The largest

* Phil. Trans. 1784.

telescope completed by him, and far exceeding in all respects any yet attempted, has an object speculum of forty feet focal length.

The reflecting telescope, which is most in use at present, is composed of two concave mirrors of different radii. The larger concave AR (fig. 113) forms the focal image im , which serves as an object for the small mirror kc : a second image im is formed by the mirror, the rays passing through the amplifying lens L , which is placed in a hole or perforation in the center of the great mirror AR . This image is erect, and is viewed much enlarged through the eye-glass or lens P .

In good reflecting telescopes the object speculum z is not of a spherical form.

Reflecting microscopes are sometimes made; the method of constructing which, as also of other instruments, may be deduced from what has been said concerning reflecting telescopes.

I N D E X

T O T H E

F I R S T V O L U M E.

A

- A***BERRATION*, 254.
Acceleration, 27.
Achromatic telescope, 351.
Aerial perspective, 346.
Air, its colour, 297. want of transparency, 345.
Algol, 192. 196.
Alps, their colour, 297.
Analogy, 2. 4.
Analysis and synthesis, 100.
Animalculæ, 12.
Aperture, 350.
Aphelium, 122.
Apogee, 122.
Appearances, mutual, of the planets, 113—115.
Apsides, 121, their motion, 232.
Archimedes, 319.
Attraction, 7. 8. 25. wherein it differs from impulse, 33. of cohesion, 44. & seq. is very properly assumed as a general cause of phenomena, 50. apparently subsisting between floating bodies, 48—50. See *Gravity*, *Cohesion*.
Atwood, Mr. his instrument for measuring the spaces described by bodies moving either uniformly or otherwise, 43.
Axiom, 2.
Axis and wheel, 60.

B

- Balance*, 59.
Block, 61.
Bradley, Dr. 240. 254.
Buffon, his burning machine, 319.
Burning mirrors, 319. glasses, 325.

Camera

I N D E X.

C

- Camera obscura*, 346.
Cause and effect, 4. 6. 50. 51.
Cause, first, is intelligent, 103.
Center, of gravity, 74—79.
Centripetal force, 94 & seq. 197 & seq.
Circumnavigation, 110.
Clocks, 88 & seq.
Clouds, why of such various colours, 297. a deception arising from their position, 346.
Cohesion, 44 & seq.
Gold, 16.
Colour, wherein it consists, 271. 272. 292 & seq.
Comets, 159—162. 167. their motion explained, 208.
Composition and resolution of motion or force, 23, 24. 62. 63.
Copernicus, 103.
Creator, 50. 111.

D

- Dead beat escapement*, 89.
Declination, 182
Degree, measure of, 236.
Demonstration, 2.
Dioptrics, 332. & seq.
Divisibility of Matter, 10—13.
Dolland, 350.

E

- Earth*, proofs of its spherical figure, 108—110. of its annual and diurnal motion, 110—112. 113—118. 236. flattened towards the poles, 235—237.
Eccentricity, 121.
Eclipse, 139. 145—158.
Ecliptic, 123. 181.
Effect and cause, 4. 6.
Elasticity, 41.
Ellipsis, its description, 97. 122.
Elongation, 105, 113—118.
Equator, 181. measure of a degree, at, 236.
Equilibrium,

I N D E X.

Equilibrium, what, 53.
Equinox, 187. precession of, 240.
Escapement in clocks, 88, 89. dead beat, 89. free, 91.
Extension, 7, 8, 9 & seq.
Eye, 259. See *Vision*.

F

Figure or form, 9.
Final causes, 162. of the planets, 164.
First cause is intelligent, 103. 163.
Fixed stars, 103. 108. have no parallax, 130. changes observed among them, 189—196. are either falling or move in immense orbits, 217. change their longitude, 240. their aberration, 256.
Focus of light, 316.
Force, or momentum, 19. accelerating or retarding, 35, 36. dispute concerning the measure of force, 37—39. centripetal, 94 & seq

G

Georgium sidus, 124.
Gold, its great ductility, 12.
Gravitation, 26 & seq. center of, 74—79. its universality deduced from its effects, 207—217.
Gravity. See *Gravitation*.

H

Halo, 280.
Halley, Dr. 132.
Hard bodies, the consequence of their stroke, 34. none are found in nature, 34.
Helix, 68, 69.
Herschel, Mr. 124. 193. 358.
Horizon, 181.

I *Incidence*,

I N D E X.

I

- Incidence*, angle of, 261.
Inclined plane, 64—66.
Inertia, 7. 8. 18.
Inflection, 298 & seq.
Intelligence finite, 1. 101.
Intuition, 2.
Impenetrability, 7. 8. 13—& seq.
Impulse, 33. is greater than pressure, 34.
Irrregularities in the planetary system, 218 & seq.
Jupiter, 118—120. its moons, 139. 314.

K

- Knowledge*, 1. 3—5. mathematical, why so perspicuous and certain, 4. philosophical, the nature of its proofs, 5. how acquired, 100.

L

- Latitude*, 182.
Lenses, 332.
Lever, 55 & seq.
Light, the quantity received by oblique surfaces, shewn, 185. in what it consists, 254. its velocity, 254. is uniform, 256. its momentum, 257. rarity, 258. various refrangibility and colour, 264 & seq. various reflexibility, 270 & seq. alternate reflection and transmission, 280 & seq. its inflection 298 & seq. powers by which its reflection and refraction are effected, 304—314.
Longitude, 182.

M

- Magic lantern*, 341.
Magnitude apparent, 335.
Maintaining power in clocks, 88.
Mars, the planet, 124. telescopic appearance, 177.
Matter, and its properties, 7 & seq.
Mathematical knowledge, why perspicuous and certain, 4. supposes extension to be analysed, 9.
Mechanical powers, enumerated, 54. their general properties, 72—74.
Medium,

- Medium*, 260. various, disperse light unequally, 269.
Mercury, the planet, 107, 176.
Micrometer screw, 71.
Microscopes, 336—340.
Mirrors, 263. 317. 355. burning, 319.
Moon, eclipse of 110. other phenomena, 141—158. telescopic appearance, 168—173. cause of its motion, 208. its irregularities explained, 218—232. colour during an eclipse, 297.
Momentum, 19. of light, 257.
Motion, 7. 8. quantity of, 19. absolute, relative and apparent, or angular, 20. laws of, 21—25. composition and resolution of, 23, 24. uniformly accelerated, 27—31. and retarded, 31—32. how produced, destroyed, or changed, 33—44. dispute concerning the measure of its quantity, 37—39.

N

- Natural philosophy*, its proofs, 5. how acquired, 100.
Nebulae, 193.
Newton, 103. 291. 315. 350. 351.
Nodes, 123.
Nutation, 240.

O

- Opacity*, its cause, 294.
Orbits, 197—207.

P

- Parallax*, annual, 120. 139. general account of, 128. horizontal, 129.
Paul's church, St. its colour at sunset, 297.
Pencil of rays, 316.
Pendulums, 26. 80. & seq. with a deal rod, 91. gridiron, 91—93. vibrate slower near the equator, 236.
Perihelium, 122.
Philosophizing, rules of, 6.
Philosophy, *natural*, its proofs, 5. how acquired, 100.
Planets, primary, 105. their mutual appearances, 113—115. their orbits elliptical, 121. nearly circular, 122. and inclined to the ecliptic, 123. their number and affections, 124—127. moons, 137. motions explained, 207. irregularities, 232.

Planets,

I N D E X.

Planets, secondary, 137. their motion explained, 208.
Porosity of matter, 17.
Precession of the equinoxes, 240.
Pressure, is less than impulse, 34.
Pringle, Sir John, 103.
Prism, 264 & seq.
Projectiles, 97.
Pulley, 61—64.

R

Radiant point, 316.
Rainbow, 273 & seq.
Reflection of light, defined, 261. analogous to refraction,
 292. regular, 317.
Reflexibility, 270.
Refraction, 261. regular, 320.
Refrangibility, 264.
Repulsion, 47. apparently exerted between floating bodies, 50.
Retarded motion, 31, 32.
Revolving bodies, 94—99.
Right ascension, 182.
Romer, Monf. 254.
Rules of philosophizing, 6.

S

Saturn and its ring, 137. 179.
Screw, 68 & seq.
Seasons, 180—188.
Shadow of the earth, 110.
Solstice, 187.
Spectacles, 333.
Speculum. See *Mirror*.
Spectrum, 266 & seq.
Stars, fixed. See *Fixed Stars*.
Sun, 104. its parallax, 131. telescopic appearance, 174. its
 atmosphere, 175.
Swing-wheel, 88.
Synthesis and analysis, 100.

T *Tackle*,

I N D E X.

T

- Tackle*, 61—64.
Telescopes, refracting, 342. their imperfections, 348. achromatic 351—353. reflecting, 358.
Termination, 183.
Tides explained, 241—251.
Transit of Venus over the Sun's disc, 132—136.
Transparency, its cause, 294.
Tropics, 188.

U

- Undulation of the air*, 345.
Universe, system of, 174.

V

- Variation of the moon's motion*, 144.
Velocity, final, 29, 31. initial, 32.
Venus, its apparent motions and proportional distance from the sun, 105—107. 113. transit over the disc of the Sun, 132—136. telescopic appearance, 176.
Vision, 258, 327 & seq.

W

- Wedge*, 66—68.
Weight, 35.
Whitehurst, Mr. his instrument for measuring the time employed by falling bodies, 42.

Y

- Year*, natural, 187. natural, periodical, and sideral, 240.

Z

- Zenith*, 182.

ID128

